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Force, Mass, and Motion

Physics 17 Laboratory
Lab #4

Abstract: We investigated the relationship between force, mass, and acceleration using force sensor carts (mass $m_c = 1.002 \pm 0.001$ kg) connected to the Science Workshop computer program. We compared measurements of the force, velocity, and acceleration of the cart as a function of time, and found that force and acceleration are directly related, while force and velocity are related only indirectly. Further experiments using carts with different masses ($m_c = 2.003 \pm 0.001$ kg, $m_c = 3.004 \pm 0.001$ kg) found that force on the cart and the acceleration of the cart are directly proportional to one another, with the mass of the cart as the constant of proportionality. These results are in excellent agreement with Newton's Second Law of Motion, F = m a.

Introduction:

In contrast to the prior Aristotelian theory of motion, which held that a force must be exerted on an object to produce a velocity, Newton's Laws of Motion state that a force exerted on an object produces only a change in velocity, *i.e.* and acceleration. In particular, the Second Law of motion states that the acceleration produced by a net force exerted on a particular object will be in the direction of the applied force, and will be proportional to the magnitude of the force, and inversely proportional to the mass of the object. This is most commonly written in the well-known form:

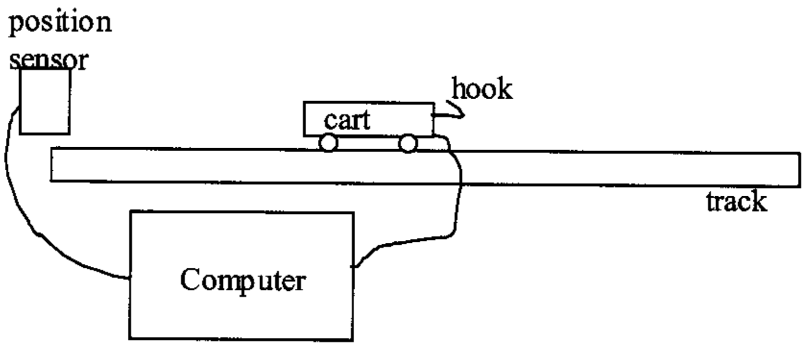
$$\mathbf{F}_{\mathbf{net}} = m\mathbf{a}$$

where the vector $\mathbf{F}_{\mathbf{net}}$ is the net force applied to the object, m is the mass, and **a** is the vector acceleration.

In this experiment, we set out to make a quantitative test of this relationship, by measuring the force applied to an object and the resulting acceleration and velocity. Force, velocity, and acceleration are all recorded by the Science Workshop software package, which allows us to make comparisons of both average forces and accelerations and also instantaneous forces and accelerations.

Experimental Procedure:

The apparatus used for this experiment consisted of a cart equipped with a force sensor connected to the Science Workshop computer interface, a track on which the cart could move back and forth in a nearly frictionless manner, and an ultrasonic position sensor, connected to the Science Workshop interface. The apparatus is shown schematically in Fig. 1:



We applied forces to the cart by grasping the hook on the force sensor cart (Fig. 1) and moving it back and forth by hand. In a preliminary experiment, we confirmed that the force sensor was reading correctly by holding the cart in place with one hand, while applying forces with the other. We also confirmed that the sign of the force recorded by Science Workshop corresponded to the direction of the force exerted on the hook; a large initial offset in the force measurement (readings of $|F|$ 7 N with no applied force) was corrected by pressing the "Tare" button on the force sensor cart.

In a first attempt to test Newton's Laws, we moved the cart back and forth along the track by means of a variable force applied to the hook. We recorded both the velocity and acceleration of the cart as it was moved back and forth, using the position sensor. Typical data are shown in Fig. 2.

We made a quantitative test of the relationship between force and acceleration by recording values of F(t) and a(t) for several times using the "smart cursor" feature of the Science Workshop package to read values from the graphs of Fig. 2. The data are presented in Table 1. Fig. 3 shows a plot of force as a function of acceleration using the data of Table 1.

A more sophisticated test of the relationship F=ma was made by repeating the above experiment, while directly recording a F vs. a plot in Science Workshop. We varied the mass of the cart by adding additional metal bars. The mass of the metal bars was found to be within 0.1% of the mass of the cart ($m_{bar} = 1.001 \pm 0.001$; $m_c = 1.002 \pm 0.001$ kg, using the electronic scale). Data for three different masses of the cart (mc = 1.002 ± 0.001 kg, 2.003 ± 0.001 kg, and 3.004 ± 0.001 kg) are plotted in Fig. 4.

Results:

While the data shown in Fig. 2 are somewhat noisy due to the errors inherent in the calculation of position and acceleration (the large fluctuations in the acceleration vs. time plot between 3.5 s and 5.3 s are the result of these errors), the qualitative relationship between force and acceleration is clear. The force and acceleration graphs are quite similar in appearance, indicating a close correlation between force and acceleration. The graph of velocity as a function of time appears quite different, indicating that there is no direct relationship between force and velocity.

The linear relationship between force and acceleration is clearly visible in Fig. 3. The solid line represents a linear fit to the data, with a slope of 0.938 kg. This differs from the measured mass of the cart (using an electronic scale), $m_c = 1.002 \pm 0.001$ kg, by 6.8%, with the difference most likely due to the large uncertainty of the acceleration as calculated by Science Workshop.

Fig. 4 clearly shows the effect of changing the mass of the cart. As the mass increases, the slope of a linear fit to the data increases proportionally. Linear fits to the data yield masses of 0.999 kg, 2.003 kg, and 2.790 kg, for the 1.002 kg, 2.003 kg, and 3.004 kg carts, respectively. The masses calculated from the fits are in good agreement with the measured mass of the cart, consistent with the expected relationship F = ma. The 7.5% difference between the gravitational and inertial mass measurements for the 3.004 kg case may be due to the increase in the frictional force ($|F_{fr}| \quad \mu_k F_N = \mu_k m_c g$, where μ_k is an effective coefficient of friction) resisting the motion of the cart along the track.

Discussion/ Conclusions:

Our experiments confirm that the acceleration produced by an applied force is indeed proportional to the magnitude of the force, as expected from Newton's Laws. The qualitative relationship between the instantaneous force and acceleration is obvious from the data of Fig. 2, and quantitative tests of the relationship were consistent with the theoretical predictions.

One possible source of systematic errors in the experiment would be frictional forces which resist the motion of the cart, but are not recorded by the force sensors. The effect of friction on the results could be tested, either by lubricating the axles of the cart to reduce the effect of friction, or by increasing the friction between the cart and the track, and measuring the effect this has on the masses obtained from linear fits to the data.

The experiments presented here have confirmed that **F** = m **a** for one-dimensional motion, however, we have not tested the full vector nature of the relationship. One potentially useful extension of this experiment would be to investigate the relationship between force and acceleration in two dimensions, by adding a second position sensor perpendicular to the first, or perhaps by using the high-speed video camera to record the motion of a force sensor cart in two dimensions.

Fig 2

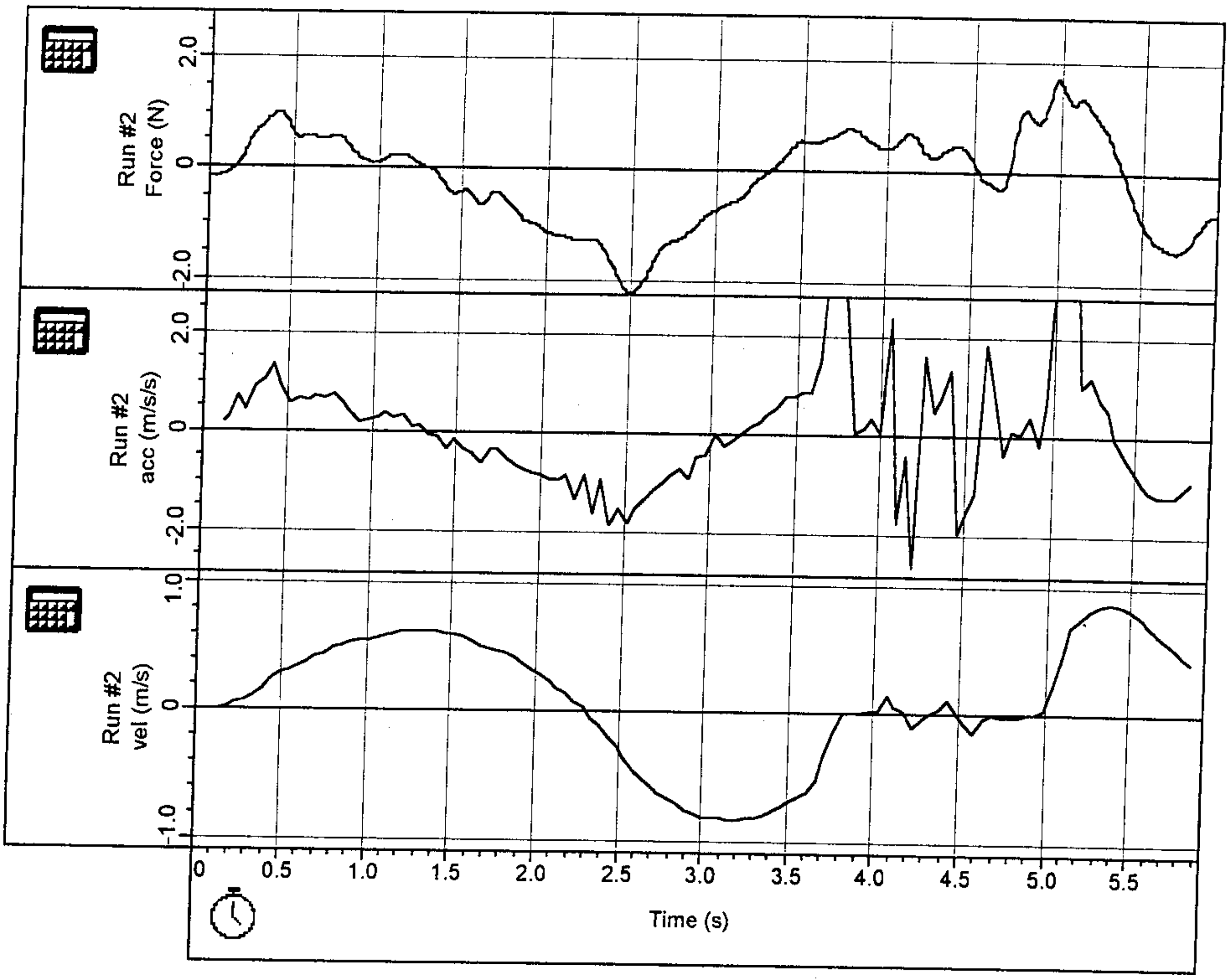

Run #2
Force (N)
2.0
0
-2.0
Run #2
acc (m/s/s)
2.0
0
-2.0
Run #2
vel (m/s)
1.0
0
-1.0
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
5.5
Time (s)

Table 1: Force and Acceleration measurements

Time (s)	Force (N)	Acceleration (m/s^2)
0.409	0.943	1.249
0.769	0.523	0.637
1.295	-0.030	-0.034
1.796	-0.740	-0.616
2.523	-2.260	-1.840

Figure 3: Force vs. Acceleration from Table 1

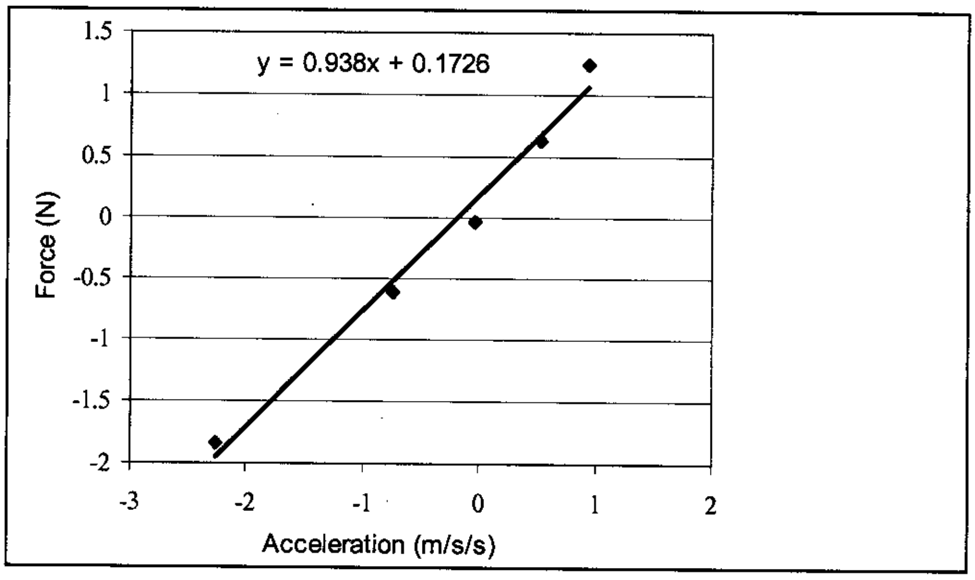
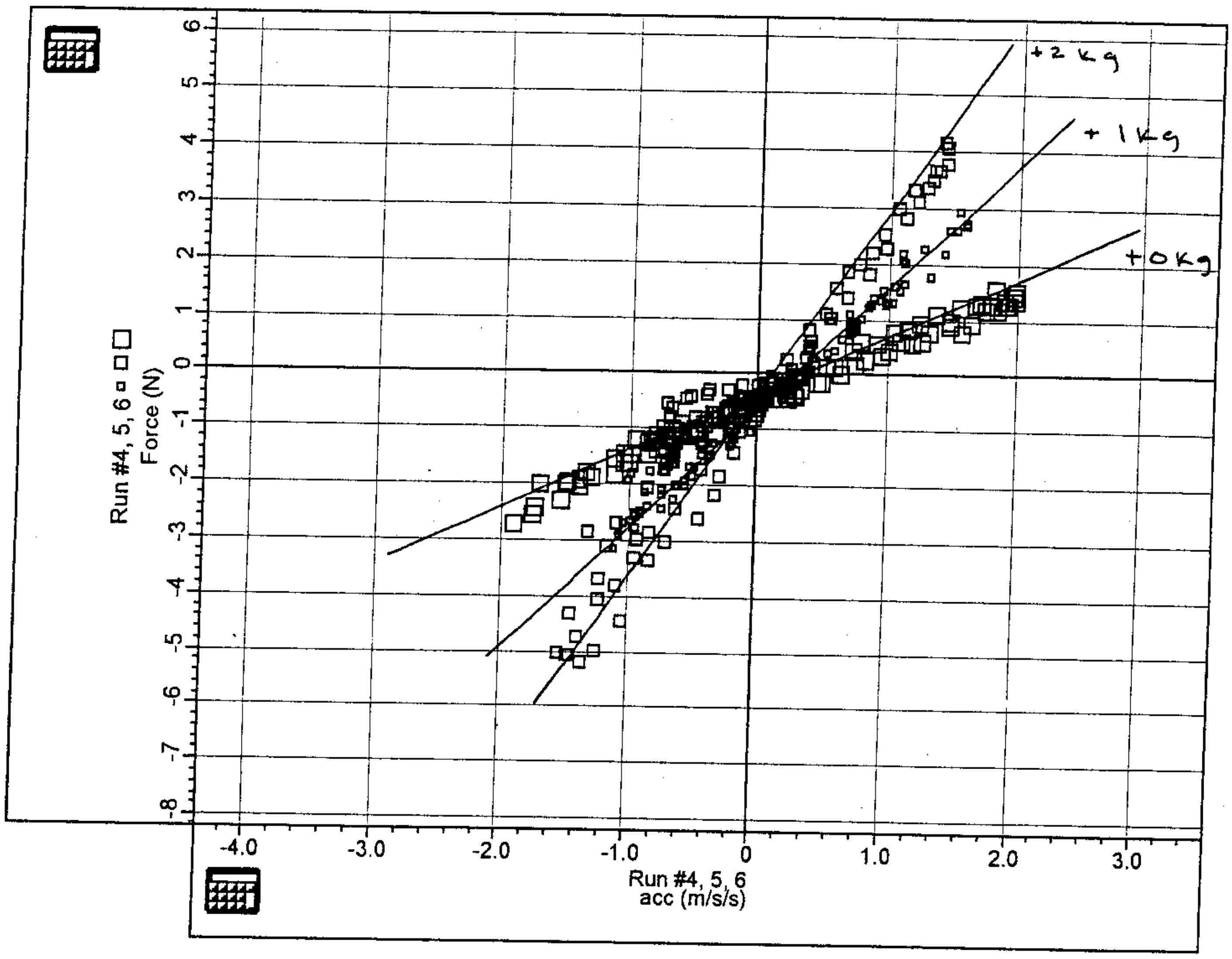


Fig 4


+2 kg
+1 kg
+0 kg
Run #4, 5, 6 ▫ □
Force (N)
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
-7
-8
-4.0
-3.0
-2.0
-1.0
0
1.0
2.0
3.0
Run #4, 5, 6
acc (m/s/s)