Asynchronous Distributed Power and Rate Control in Ad Hoc Networks with Stochastic Channels

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***Abstract*— This paper analyzes distributed asynchronous power and rate control for wireless ad hoc networks with stochastic channels. In contrast to conventional cellular systems, all network transmitters are assumed to be independent of any management infrastructure and, importantly, to have the freedom to choose their *own* arbitrary control rules, using as input only the information on local interference and achieved signal-to-interference and noise ratio ($SINR$). This approach respects link's different local network conditions and preferences on quality of service. With the purpose of finding network-wide acceptable equilibria for such an individually defined power/rate allocation dynamics, we discuss an entirely general asynchronous and distributed algorithm, whereby stochastic channels are assumed. Moreover, optimum admission scheme for linear/linearized models is given. Numerical simulations show the efficiency of our approach to allocate comparably higher $SINR$s in random ad hoc networks with changing topologies and user density.**

***Index Terms*— Ad hoc networks, power and rate control, distributed, asynchronous, equilibrium, convergence.**

I. I

The concept of designing wireless communication networks without a centralized aid of backbone controllers (cell base stations), which facilitate and carry out network management on a local cell level, is attractive from the economical and practical point of view. The interest in such ad hoc networks stems among others from their ability to provide flexible, rapidly deployable and fully distributed wireless networking [1] such as in disaster rescue or military applications.

The choice for independence from expensive and slowly deployable vulnerable base stations however requires their substitution by distributed control algorithms, carried out by network users *themselves*. Preferably not only distributed, but also asynchronous control mechanisms are needed in view of difficulties with achieving a network-wide synchronization.

This paper proposes a new approach to the classical power and rate control problem, whereby we assume the challenging context of above introduced ad hoc networks, requiring its implementation by means of collective decision-making. What makes this a difficult task is besides the need for distributivity and asynchrony also the fact that communication channels have a stochastic nature and ad hoc network users are generally non-cooperative and selfish. The latter commonplace assumption suggests users' tendency to use higher power outputs to overcome experienced co-channel interference from others and maximize their immediate quality of service (QoS) profits disregarding however the harms caused by one's own power.

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II. R W S R

Cutting this vicious circle, which ultimately degrades the performance of the entire ad hoc network, calls for algorithmic rules for power/rate control, which would be beneficial to all and hence persuade rational network users to follow them.

One approach to this issue consists in solving problems of maximizing a minimum $SINR$ ratio [2] or satisfying target $SINR$ levels with a minimum total transmit power [3]. Recent works inspired e.g. by [4] have then tried to apply game-theoretic QoS maximization to the field of dynamic optimization of resources allocation and to solve among others the possible divergence of early algorithms. These works, however, often depart from strongly application dependent system models and not surprisingly offer results with low mathematical generality or practical universality.

In this paper, we develop a novel "best-response" approach to distributed power and rate control for multihop ad hoc communication systems supporting several frequency bands and having stochastic channels. In our model, each active link (i.e., independent single-hop transmission) adjusts periodically and asynchronously its transmit power and rate, determining their values using respectively *best-response* and *transmit rate assignment* functions with the information on local co-channel interference and achieved $SINR$ as inputs. Importantly, we leave up to links themselves to design the particular realization of these update functions according to their (slowly) varying preferences or needs in order to match variable service requirements and changing local network conditions.

As for novelty, the present paper develops our own prior work [5], which we thus consider as the closest reference. Therein, we have studied *power* control with *invariable* best-response functions from a *game-theoretic* point of view assuming *deterministic* channel gains. In this contribution, we present a more complex case of *power and rate* control with possibly *slowly varying* best-response and rate assignment functions using *stochastic approximation* to deal with the effects of *random* channels gains. Nevertheless, our work is done on a *purely* theoretical level and focuses on analytical clarity and simplicity without making any technical assumption, which further delimits it from other prior publications.

The following text first defines in Section III our power/rate control model and then discusses in Section IV the problem of the convergence of its dynamics to Nash equilibria assuming stochastic channels. In Section V, we focus more on linear (linearized) control system model and present as a result an optimum admission control scheme for this case. Numerical simulations are given in Section VI, followed by conclusion.

III. S M

When analyzing a general wireless ad hoc network using data relaying over multihop connections on multiple frequency bands, we employ a decomposition of the network into orthogonal frequency bands [6] and study with no loss of generality only the reduced problem of isolated resources management in a single channel network (CDMA or FDMA) with concurrent interference, caused by N simultaneously active links. Let $\left[h_{ij}\right]$ be the $N \times N$ channel gain matrix, whereby h_{ij} is the channel gain from the transmitter of link j to the receiver of link i.

We think of power/rate control as of individual decision-making in a non-cooperative environment, whereby each network link i performs its power/rate control updates periodically with period T in time instances t_i^k for integer k. We generally assume an asynchronous power/rate control with $t_i^k \neq t_j^k$ for all links i and $j \neq i$ (other links than link i are denoted by $-i$).

Links adjust their transmit powers σ_i with respect to experienced co-channel interference using their so-called best-response functions $\beta_i^{\mathrm{RX}} \geq 0$ [5], describing mathematically the value of the most desirable received power $h_{ii}\sigma_i$ at link i's receiver (RX) necessary to overcome a given interference $\sum_{j\neq i} h_{ij}\sigma_j = \boldsymbol{h}_{-i}^{\mathrm{T}}\boldsymbol{\sigma}_{-i}$ from other interfering links $j \neq i$ $(-i)$ in order to achieve an acceptable connection, e.g., an acceptable $SINR_i$. Boldface notation denotes vectors and matrices.

As individual links may possibly have different preferences on $SINR_i$ for considered interference values, each link defines its own function β_i^{RX} independently from others and based solely on its own individual preferences or QoS needs. Functions β_i^{RX} are therefore generally different among each other.

Link i's transmitter (TX) adjusts concurrently with its transmit power also its transmit data rate ϱ_i based on a so-called rate assignment function $R_i^{\mathrm{TX}} \geq 0$, which assigns preferred data rates to $SINR_i$ achieved at the receiver. Likewise β_i^{RX}, R_i^{TX} is assumed to be link i's particular choice too.

We formulate similarly to [5] the above distributed power/rate control model as a strategic game in order to incorporate into our model the fact that in the above scheme all network links take independently their best available actions (power/rate updates with respect to β_i^{RX} and R_i^{TX}) to pursue their own individual objectives (choice of β_i^{RX} and R_i^{TX}) using only the information on mutual interference and achieved $SINR_i$. What makes this a strategic game is that what is best for one link, in general, depends upon other links' actions.

Setting $\beta_i^{\mathrm{TX}}(\boldsymbol{\sigma}_{-i}) \overset{\mathrm{def}}{=} \frac{1}{h_{ii}}\beta_i^{\mathrm{RX}}\left(\boldsymbol{h}_{-i}^{\mathrm{T}}\boldsymbol{\sigma}_{-i}\right)$ to be the transmitter TX based best-response function, we formally redefine our system model as a non-cooperative game $\mathscr{G} = \{\mathscr{I}, \sigma_i, \beta_i\}$ [7] with these three elements (in the next we omit for clarity TX in β_i^{TX} and R_i^{TX} and if possible also the ubiquitous t_i^k in $\sigma_i^{t_i^k}$):

1) a set $\mathscr{I}$ of N active links $i \in \mathscr{I}$, $\mathscr{I} = \{1, ..., N\}$ (game players), simultaneously accessing the shared medium;
2) a transmit power $\sigma \in \Sigma_i$ (game strategy) for each link i, where $\Sigma_i \in \Re^{1+}$ defines the power profile of link i;
3) link i's best-response function $\beta_i : \boldsymbol{\Sigma}_{-i} \to \Sigma_i$ (transmit power update rule), assigning a power $\sigma_i \in \Sigma_i$ to each opponent links play $\boldsymbol{\sigma}_{-i} \in \boldsymbol{\Sigma}_{-i}$;

wherein $\boldsymbol{\Sigma}_{-i}$ with elements $\boldsymbol{\sigma}_{-i}$ represents the strategy profile of opponent links $-i$, given by the Cartesian product of $\boldsymbol{\Sigma}_j$ for all $j \in \mathscr{I}$ and $j \neq i$. $\boldsymbol{\Sigma}$ with elements $\boldsymbol{\sigma}$ denotes the strategy profile of the entire power control game, given by the Cartesian product of $\boldsymbol{\Sigma}_i$ for all $i \in \mathscr{I}$.

Note that transmit power σ_i is the only strategy in our game-theoretic model formulation, because knowing all powers σ_i and functions R_i is sufficient to uniquely derive links' $SINR$s and thus their corresponding data rates. Reaching a network-wide equilibrium by the power control process therefore implies also a stabilization of the rate adjustment process.

IV. S N E U S C

The main contribution of our work consists in analyzing how to distributively and asynchronously find suitable solutions to the game $\mathscr{G}$ under the real-life assumption of stochastic communication channels h_{ij}. It is certainly natural to require such solutions to be stable power vectors $\hat{\boldsymbol{\sigma}}$ allocated by individual links such that every link i's transmit power is optimally adapted to interference from other links with respect to given β_i, i.e., $\hat{\sigma}_i = \beta_i(\hat{\boldsymbol{\sigma}}_{-i})$ for each link. Taking an insight of game-theory, this formulation in fact mathematically corresponds to the definition of Nash equilibrium [7].

In order to be able to use the Nash equilibrium concept within the discussed power/rate control framework, we must assume two mild restrictions on $\boldsymbol{\Sigma}$ and β_i. Let for all $i \in \mathscr{I}$ (i) each link's i strategy profile Σ_i be a non-empty, compact and convex set of real numbers $\Re^1$ and (ii) β_i be a globally Lipschitz in the strategy space $\boldsymbol{\Sigma}$ (bounded first derivative). The motivation for this is that one can show using Brouwer's fixed point theorem [8] that under such assumptions the game $\mathscr{G}$ *always* admits at least one Nash equilibrium $\hat{\boldsymbol{\sigma}} \in \boldsymbol{\Sigma}$.

So solving $\mathscr{G}$ boils down to a distributed and asynchronous search for zero root(s) $\hat{\boldsymbol{\sigma}}$ of the function $\boldsymbol{f}(\boldsymbol{\sigma}) = \boldsymbol{\beta}(\boldsymbol{\sigma}) - \boldsymbol{\sigma}$ having available only periodical observations of interference, i.e., $\boldsymbol{\beta}(\boldsymbol{\sigma})$, where $\boldsymbol{\beta}$ is the Cartesian product of β_i on $\boldsymbol{\Sigma}$. Yet under the presence of noise ϵ in channels, $\boldsymbol{\beta}(\boldsymbol{\sigma})$ becomes noisy, which accordingly results into noisy observations $[\boldsymbol{f}(\boldsymbol{\sigma}) + \epsilon]$ of $\boldsymbol{f}(\boldsymbol{\sigma})$. To mitigate this problem, a recursive *synchronous* algorithm (time index k instead of t_i^k) has been proposed in [9] to find a sought-for root $\hat{\boldsymbol{\sigma}}$ under stochastic conditions:

$$\boldsymbol{\sigma}^{k+1} = \boldsymbol{\sigma}^k + a^k \left[\boldsymbol{f}\left(\boldsymbol{\sigma}^k\right) + \epsilon^k\right], \qquad (1)$$

where a^k is the algorithmic step size. This algorithm has been used for example in [3] to improve a poor performance of the classical Foschini-Miljevic algorithm in noisy channels, which allocates minimum transmit powers for satisfying hard constraints on $SINR_i$.

In our work however, we have alternatively focused on modifying a more elaborate stochastic approximation algorithm from [10] to fit to our specific purposes. As a result, the next paragraph presents an algorithm that not only allows for a distributive search for an equilibrium $\hat{\boldsymbol{\sigma}}$ under noise as in the case of Eq. (1), but also solves the problem of *asynchronous* convergence to potentially *multiple* equilibria of $\mathscr{G}$, weakens noise conditions compared to [3] and importantly allows an iteration *restart* if the estimate of $\hat{\boldsymbol{\sigma}}$ departs out of $\boldsymbol{\Sigma}$.

Suppose periodical, but asynchronous transmit power updates as defined in the system model. Then the proposed algorithm consists in letting every link perform its power update actions at time instances t_i^k based on the following recursive formula, whose only input is the information on interference from asynchronously updated transmit powers $\boldsymbol{\sigma}_{-i}^{t_i^k}$ and which is defined for some initial transmit power $\sigma_i^{t_i^0} \in \Sigma_i$:

$$\sigma_i^{t_i^k+T} = \left(\sigma_i^{t_i^k} + a_i^k y_i^{t_i^k}\right) \mathrm{L}_{\left[\left(\sigma_i^{t_i^k} + a_i^k y_i^{t_i^k}\right) \in \Sigma_i\right]} + \sigma_i^* \mathrm{L}_{\left[\left(\sigma_i^{t_i^k} + a_i^k y_i^{t_i^k}\right) \notin \Sigma_i\right]} \quad (2)$$

The term $y_i^{t_i^k} = f_i\left(\boldsymbol{\sigma}_{-i}^{t_i^k}\right) + \epsilon_i^{t_i^k}$ represents noisy observations of the function $f_i = \beta_i\left(\boldsymbol{\sigma}_{-i}^{t_i^k}\right) - \sigma_i^{t_i^k}$ having Nash equilibrium (equilibria) as its zero root(s). The logical function $\mathrm{L}_{[statement]}$ in Eq. (2) is equal to 1 if its *statement* is true and becomes 0 otherwise, whereby if one candidate value $\sigma_i^{t_i^k+T} = \sigma_i^{t_i^k} + a_i^k y_i^{t_i^k}$ of the estimated equilibrium component $\hat{\sigma}_i$ at time $t_i^k + T$ exits from the predefined transmit power profile Σ_i, the search for the equilibrium $\hat{\boldsymbol{\sigma}}$ is generally pulled back to the pre-specified point $\boldsymbol{\sigma}^* \in \boldsymbol{\Sigma}$ and restarted from this new initial value.

In order to describe the convergence conditions of Eq. (2), we must make the following assumption on noise ϵ.

Assumption 1: For any convergent subsequence $\{\sigma_i^{t_i^n}\}$ of $\{\sigma_i^{t_i^k}\}$, any $i \in \mathscr{I}$ and any $\theta_k \in [0, \theta]$

$$\limsup_{\theta \to 0} \limsup_{k \to \infty} \frac{1}{\theta} \left| \sum_{s=n}^{m(n,\theta_k) \wedge r\left(i, g_i^n + 1\right)} a_i^s \epsilon_i^{t_i^s} \right| = 0, \quad (3)$$

where "$\wedge$" denotes the minimum between $m(n, \theta_k) = \inf\{l \geq n, \sum_{s=n}^{l} a_i^s > \theta_k\}$ and $r(i, l) = \inf\{k > 0, g_i^k = l\}$, whereby g_i^k is a virtual truncations counter $g_i^{k+1} = g_i^k + \mathrm{L}_{\left[\left(\sigma_i^{t_i^k} + a_i^k y_i^{t_i^k}\right) \notin \Sigma_i\right]}$ with $g_i^0 = 0\ \forall i$. □ Then we state:

Theorem 1: Assume a non-cooperative power control game $\mathscr{G} = \{\mathscr{I}; \sigma_i; \beta_i\}$ as defined in the system model and let $\boldsymbol{\beta}$ be globally Lipschitz functions $\boldsymbol{\beta} : \boldsymbol{\Sigma} \to \boldsymbol{\Sigma}$ in a non-empty compact and convex strategy space $\boldsymbol{\Sigma}$, whereby the set $J = \{\hat{\boldsymbol{\sigma}} \in \boldsymbol{\Sigma} : \boldsymbol{f}(\hat{\boldsymbol{\sigma}}) = \boldsymbol{\beta}(\hat{\boldsymbol{\sigma}}) - \hat{\boldsymbol{\sigma}} = 0\}$ denotes a set of isolated Nash equilibria $\hat{\boldsymbol{\sigma}}$ of $\mathscr{G}$ for given $\boldsymbol{f}$, resp. $\boldsymbol{\beta}$. Let links' transmit powers $\sigma_i^{t_i^k}$ be periodically updated at time instances t_i^k based on Eq. (2) with an initial value $\sigma_i^{t_i^0}$ and for $\sigma_i^* < \max[\Sigma_i]$, whereby each link iterates Eq. (2) with its own step-size a_i^k such that $a_i^k > 0$, $a_i^k \xrightarrow{k\to\infty} 0$, $\sum_{k=0}^{\infty} a_i^k = \infty$ and

$$0 < d_i^{\min} \leq \liminf_{k\to\infty} \frac{a_i^k}{a_{j\neq i}^k} \leq \limsup_{k\to\infty} \frac{a_i^k}{a_{j\neq i}^k} \leq d_i^{\max} \quad a.s. \quad (4)$$

for some $d_i^{\min}$ and $d_i^{\max}$. Then if there exist scalar twice continuously differentiable function $v : \boldsymbol{\Sigma} \to \mathfrak{R}^1$ such that

$$\sup_{d_i \in \left[d_i^{\min}, d_i^{\max}\right]} \left[\boldsymbol{f}^{\mathrm{T}} \operatorname{diag}(1, d_2, \ldots, d_N) \operatorname{grad} v\right] < 0, \quad (5)$$

for all $\boldsymbol{\sigma} \in \boldsymbol{\Sigma} \setminus J$ and $v(\boldsymbol{\sigma}^*) < \inf_{\boldsymbol{\sigma}:\sigma_i = \max[\Sigma_i];\, \sigma_j \leq \max\left[\Sigma_{j\neq i}\right]} v(\boldsymbol{\sigma})$ is true together with the *Assumption 1* on noise ϵ_i, it holds that

$$\operatorname{dist}\left[\boldsymbol{\sigma}^{t_i^k}, J\right] \xrightarrow{k\to\infty} 0 \quad a.s. \quad (6)$$

for $\operatorname{dist}\left[\boldsymbol{\sigma}^{t_i^k}, J\right] = \inf\{\|\boldsymbol{\sigma}^{t_i^k} - \hat{\boldsymbol{\sigma}}\|\ \forall \hat{\boldsymbol{\sigma}} \in J\}$. □[1]

[1]The proofs of the theorems stated herein are provided in the extended journal version of the paper.

Apart of verifying the noise assumption for some particular channel type of interest, all the other assumptions of the above theorem are not very complicated. Importantly, we observe that the function v is *not* required to be non-negative as in the case of the Lyapunov function V, needed for assuring power control stability in [5]. So a candidate function for v can be found e.g. by means of a function $g_i = \operatorname{grad}_i v$ for all i such that (i) $f_i g_i < 0$ and (ii) $\left[\frac{\partial g_i}{\partial \sigma_j}\right]$ is symmetric (recall that $\boldsymbol{g}$ is a gradient of a scalar function v if and only if $\boldsymbol{g}$'s Jacobian matrix is symmetric). In contrary to [5], it is not necessary to search for $\boldsymbol{g}$ such that it also holds that (iii) $v = \int_0^{\boldsymbol{\sigma}} \boldsymbol{g}(\boldsymbol{\tau})\, d\boldsymbol{\tau} > 0$.

V. P R C L L B -R F

While this method for generating functions v is straightforward and applicable to general power control, it may not be always practical as the analytical form of β_i can be complicated or defined numerically. Let us therefore restrict our attention to a simplified system model, whose best-response functions β_i are assumed to be linear (linearized) functions in the form $\beta_i = \left(A_i + B_i \boldsymbol{h}_{-i}^{\mathrm{T}} \boldsymbol{\sigma}_{-i}\right)/h_{ii}$ for $A_i, B_i \in \mathfrak{R}^1$. We naturally assume continuous differentiability of $\boldsymbol{\beta}$ at equilibrium $\hat{\boldsymbol{\sigma}}$ in the case of linearizing a nonlinear power control dynamics around $\hat{\boldsymbol{\sigma}}$ with approximate linear functions having slopes $B_i' = \frac{\partial \beta_i(\boldsymbol{\sigma})}{\partial\left(\boldsymbol{h}_{-i}^{\mathrm{T}} \boldsymbol{\sigma}_{-i}\right)}$.

Two important power/rate control schemes can be modeled in such a way. First, if $A_i > 0$ and $B_i < 0$, then each link's preferences on $SINR_i$ decrease inverse proportionally with increasing interference. In such a case, links can take advantage of high-power high-speed transmissions for lower interference levels and use rather slow, but energy saving transmissions for higher interference. If interference exceeds the threshold A_i/B_i, link i becomes passive and waits for better channel conditions while saving its power budget (this implements a receiver-based admission control of CSMA/CA kind). Secondly, if $A_i = 0$ and $B_i > 0$, then the system model represents a network with hard constraints on $SINR$ [12].

Defining matrix $\boldsymbol{B} = \left[L_{[i\neq j]}\ B_i\ h_{ij}\ /\ h_{ii}\right]$, we assume $\boldsymbol{B}$ to be a nonsingular and irreducible matrix, since different active links have disjoint positions in the network. Assuming for simplicity all $h_{ij} = const$ as a result of interference data averaging and/or slow mobility, we can state the next theorem:

Theorem 2: Assume in accordance with the system model a non-cooperative power control game $\mathscr{G} = \{\mathscr{I}; \sigma_i; \beta_i\}$ having linear or linearized $\beta_i = \left(A_i + B_i \boldsymbol{h}_{-i}^{\mathrm{T}} \boldsymbol{\sigma}_{-i}\right)/h_{ii}$ for $A_i, B_i \in \mathfrak{R}^1$ and $\forall i \in \mathscr{I}$. Then the game $\mathscr{G}$ has a unique Nash equilibrium $\hat{\boldsymbol{\sigma}}$ and its best-response power control dynamics

1) converges globally asymptotically to $\hat{\boldsymbol{\sigma}}$;
2) diverges globally from $\hat{\boldsymbol{\sigma}}$;

if it holds that

1) $h_{ii} > |B_i| \sum_{j\neq i} h_{ij}$ for all i;
2) $h_{ii} < |B_i| \sum_{j\neq i} h_{ij}$ for some i or at least one $\boldsymbol{B}$'s eigenvalue $|\lambda_i(\boldsymbol{B})| = 1$ corresponds to a Jordan cell with dimension more than 1;

respectively, whereby the speed of convergence of Eq. (2) in case (1) or its divergence in case (2) is exponential. □

This theorem in fact defines an optimum admission control scheme on the data link layer, which is cross-layer optimized with the power and rate control of the underlying physical layer in terms of assuring its network-wide convergence. Moreover, it supplements the admission control implicit to our system model and characterized by the interference threshold A_i/B_i. Note also the theorem's relation to the network layer through the parameter A_i, which can be adaptively adjusted for networking and routing purposes so that link i's transmit power is allocated correspondingly to its geometrical length.

Evidently, using *Theorem 2*, each link can determine *independently* from others, whether its transmission, under given channel conditions h_{ii} and $\boldsymbol{h}_{-i}$ and with a best-response function characterized by A_i and B_i, would affect the convergence process of the whole network. Advantageously, all of these parameters are *locally* available information for link i.

Following the admission rule of case (1) by all links assures that the power/rate control converges globally and asymptotically with an exponential motion to a unique Nash equilibrium for any initial condition. In contrary, a transmission violating the admission control as in case (2) results into an undesirable exponential divergence of transmit power allocation.

In order to propose similarly to *Theorem 1* an asynchronous and distributed solution to *noisy* power/rate control ($h_{ij} \neq$ *const*) with linear best-response functions, we have further developed *Theorem 2* and showed that implementing its admission control scheme is in fact equivalent to assuring convergence conditions of Eq. (2). Although the proof requires several consecutive steps, it is not very hard to derive the following substantially simplified version of *Theorem 1*:

Theorem 3: Assume a non-cooperative power control game $\mathcal{G} = \{\mathcal{I}; \sigma_i; \beta_i\}$ as defined in the system model having linear or linearized $\beta_i = \left(A_i + B_i \boldsymbol{h}_{-i}^{\mathrm{T}} \boldsymbol{\sigma}_{-i}\right)/h_{ii}$ for $A_i, B_i \in \Re^1$ and $\forall i \in \mathcal{I}$. Let each link update its transmit power $\sigma_i^{t_i^k}$ at time instances t_i^k based on Eq. (2) with an initial value $\sigma_i^{t_i^0}$ and for $\sigma_i^* < \max[\Sigma_i]$ using its own step-size a_i^k such that $a_i^k > 0$, $a_i^k \xrightarrow{k\to\infty} 0$, $\sum_{k=0}^{\infty} a_i^k = \infty$ and

$$0 < d_i^{\min} \leq \liminf_{k\to\infty} \frac{a_i^k}{a_{j\neq i}^k} \leq \limsup_{k\to\infty} \frac{a_i^k}{a_{j\neq i}^k} \leq d_i^{\max} \quad a.s. \qquad (7)$$

for some $d_i^{\min}$ and $d_i^{\max}$. Then if $h_{ii} > |B_i| \sum_{j\neq i} h_{ij}$ for all i and the *Assumption 1* on noise ϵ_i is true, it holds that $\|\boldsymbol{\sigma}^{t_i^k} - \hat{\boldsymbol{\sigma}}\| \xrightarrow{k\to\infty} 0 \quad a.s.$, whereby $\hat{\boldsymbol{\sigma}}$ is the unique equilibrium of $\mathcal{G}$. □

By the nature of stochastic approximation, the particular choice of the algorithmic step-size a_i^k plays a key role in a well-known tradeoff between the algorithmic convergence rate and error distance of the allocated power vector from the optimum equilibrium vector. A step-size with bigger magnitude would in general imply faster convergence with lower precision of the equilibrium estimate, whereby a smaller magnitude step-size allows reaching the equilibrium without larger oscillations around it, but the convergence rate would be accordingly slower. Detailed numerical simulations, related to said step-size choice trade-off, can be found e.g. in [13].

In this context [3] also proposes to use a constant step-size in order be able to track down time-varying equilibria and respond to admission events in the network. Such an approach however offers only convergence in distribution.

VI. N R

In the next, we show that the proposed approach to power and rate control has the potential to allocate comparably better $SINR$s in large and densely populated ad hoc networks. The focus on $SINR$ is necessary, because our approach to power/rate control is driven solely by adaptation to local co-channel interference, whereas many fundamental formulas of the information theory depend on $SINR$, i.e., on the *ratio* of the received power $h_{ii}\sigma_i$ and the actual interference $\boldsymbol{h}_{-i}^{\mathrm{T}}\boldsymbol{\sigma}_{-i}$.

Assume a square area of 10 km × 10 km with the number of stationary links varying from 1 to 100, whereby their length is set to 100 m in order to enhance the interpretability of the following simulations (results hold also for the general case). All links update in an asynchronous and distributive way their powers using for simplicity the same linear best-response functions $\beta_i(\boldsymbol{\sigma}_{-i}) = h_{ii}P^{\max} - \boldsymbol{h}_{-i}^{\mathrm{T}}\boldsymbol{\sigma}_{-i}$, whereby the network maximum transmit power $P^{\max}$ is set to 1 W. This scheme corresponds to the usage of "bursty" high-speed high-power transmissions in low interference conditions and vice versa as discussed at the beginning of Section V. Channels gains h_{ij} exhibit a path loss with exponent $\alpha = 3.5$. Optimum admission control from *Theorem 2* is used to assure stability.

We compare equilibrium $SINR$s of this "proposed algorithm", representing our framework, with $SINR$s allocated by a "constant received power" algorithm, inspired vaguely by [14]. This comparison algorithm (i) allocates every link's transmit power such that all receivers receive the *same* power, whereby for a precisely-defined and fair comparison (ii) said equal received powers are such that the sum of all transmit powers used by the comparison algorithm is equal to the sum of all equilibrium powers, allocated by the proposed algorithm.

In the first simulation scenario, we vary the total number of active and uniformly randomly distributed links in the network, starting from 1 and ending by 100, and compare the performance of both said algorithms in terms of allocated $SINR$s. Fig. 1 depicts histograms of maximum achieved $SINR$ differences between the proposed algorithm and the comparison one, whereby each curve represents a different number of active links.

We observe that the growing link density in the network (and thus worse interference conditions) implies a shift of the differential histogram curves in the direction of *positive* values, which means an increasing *improvement* of $SINR$s by the proposed algorithm with respect to the comparison algorithm under worsening interference conditions.

The observed large improvement difference between sparse and dense networks lies in the fact that in rather sparse networks the interference can be significantly heterogeneous. Simulated *same* best-response functions then allocate mutually more different powers, which lays the ground for a significant local $SINR$ improvements. Whereas in denser networks, the interference is more homogenous and allocated powers comparable. Consequently $SINR$s are also more balanced, yielding finally only lower improvement possibilities.

Our second simulation scenario evaluates, how the proposed algorithm can improve links' $SINR$s if the randomness of simulated network topologies varies. We simulate this change

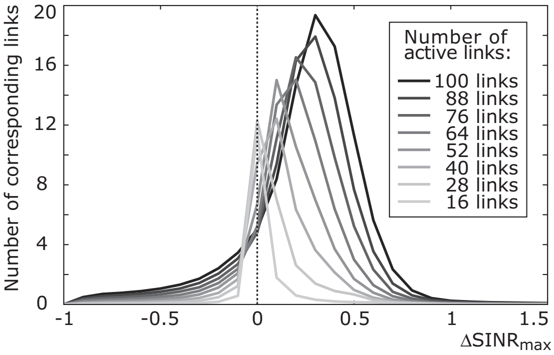

Fig. 1. Histogram of maximum achieved $SINR$ differences between the proposed and comparison algorithms. Eight cases of increasing number of active links in the network are illustrated, each obtained from over 10^4 random topologies.

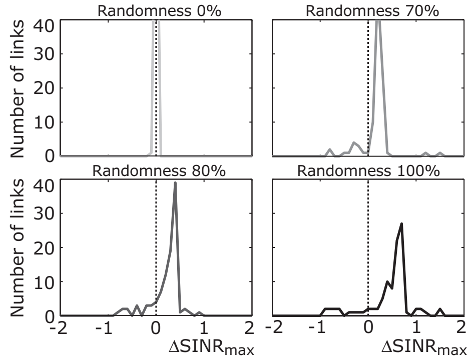

Fig. 2. Histogram of maximum achieved $SINR$ differences between the proposed and comparison algorithms. Four cases of increasing topological randomness in a network with 100 active links are shown, each obtained from over 10^4 random topologies.

as a continuous transition from regular grid-like topologies (0% randomness) to uniformly randomly distributed topologies (100% randomness). More precisely, we imagine a regular grid with 10 rows and 10 columns superposed over the whole network area, whereby the geometric center points of our 100 links are positioned uniformly randomly into virtual square deviation areas centered on the grid's intersections. During a simulation run the deviation area size is progressively increased, starting from zero (only the grid intersections with zero surface are comprised) and ending by a case, allowing positioning of links anywhere into the whole network area and generating thus fully uniformly random topologies.

Fig. 2 shows four histograms of maximum achieved $SINR$s differences between the proposed algorithm and the comparison one for four representative levels of topological randomness - 0%, 70%, 80% and 100%. We can again observe a progressive shift of $SINR$ differences to the positive side of each histogram' x-axis in response to increasing topological randomness, which proves a better comparative $SINR$ allocation performance of the proposed algorithm in random topologies.

Both algorithms allocate practically the same $SINR$s in regular topologies with homogenous grid-like layout (see the zero centered $SINR$ differences for 0% randomness in Fig. 2). This is due to the fact that all links have the same best-response functions and the proposed algorithm thus allocates to each link the same transmit power reflecting a homogenous interference between equidistant links (the influence of their angular orientations averages out). When the network topology starts to be more random, the average network $SINR$ improves similarly to the first simulation scenario.

The above data therefore represent a comprehensible evidence that our concept of adaptively adjusting transmit powers based on mutual interference is more advantageous in terms of allocated $SINR$ than the compared strategy of maintaining a constant received power.

VII. C

In this work, we studied distributed asynchronous power and rate control for ad hoc networks with stochastic channels using general best-response and rate assignment functions from a game-theoretical point of view. Restricting our model by only minimal necessary mathematical assumptions, we showed conditions for convergence of such a power and rate control dynamics to Nash equilibria, whereby our analysis is entirely general and application independent. In addition, optimum admission control scheme for linear/linearized control models was shown as an application of our framework, together with proving the potential of our approach to provide for a satisfactory and comparably better $SINR$ allocation by means of numerical simulations. Our current research includes modifying the above framework for modeling power and rate control in channels with state-dependent noise.

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[1] A. J. Goldsmith and S. B. Wicker, "Design challenges for energy-constrained ad hoc wireless networks," *IEEE Trans. Wireless Commun.*, vol. 9, no. 4, pp. 8–27, Aug. 2002.

[2] J. Zander, "Distrib. co-channel interf. control in cellular radio systems," *IEEE Trans. Veh. Technol.*, vol. 41, no. 4, pp. 305–311, Aug. 1992.

[3] T. Holliday, A. Goldsmith, P. Glynn, and N. Bambos, "Distributed power and admission control for time-varying wireless networks," in *Proc. of IEEE Globecom*, Dallas, Texas, Nov. 29 – Dec. 3 2004, pp. 768–774.

[4] C. U. Saraydar, N. B. Mandayam, and D. J. Goodman, "Efficient power control via pricing in wireless data networks," *IEEE Trans. Commun.*, vol. 50, no. 2, pp. 291–303, Feb. 2002.

[5] S. Kucera, K. Yamamoto, and S. Yoshida, "Distributed power control for wireless ad hoc networks: A game-theoretic approach based on best-response functions," in *Proc. of IEEE VTC Fall 2006*, Montreal, Canada, Sep. 25 – 28 2006.

[6] N. Bambos, "Toward power-sensitive network architectures in wireless communications: Concepts, issues, and design aspects," *IEEE Personal Commun. Mag.*, vol. 5, no. 3, pp. 50–59, June 1998.

[7] D. Fudenberg and J. Tirole, *Game Theory*. MIT Press, 1991.

[8] G. Debreu, *Theory of Value: An Axiomatic Analysis of Economic Equilibrium*. New York: John Wiley & Sons, 1959.

[9] H. Robbins and S. Monro, "A stochastic approximation method," *Annals of Mathematical Statistics*, vol. 22, no. 3, pp. 400–407, 1951.

[10] H.-F. Chen, *Stochastic Approximation and Its Applications*. Dordrecht, The Netherlands: Kluwer Academic Publishers, 2002.

[11] D. Schultz and J. E. Gibson, "Variable gradient method for generating Lyapunov functions," *AIEE Transactions*, vol. 81, pp. 203–209, 1962.

[12] G. Foschini and Z. Miljanic, "A simple distributed autonomous power control algorithm and its convergence," *IEEE Trans. Veh. Technol.*, vol. 42, no. 4, pp. 641–646, Nov. 1993.

[13] S. Ulukus and R. D. Yates, "Stochastic power control for cellular radio systems," *IEEE Trans. Commun.*, vol. 46, no. 6, pp. 784–798, June 1998.

[14] W. Yu and J. M. Cioffi, "Constant-power waterfilling: performance bound and low-complexity implementation," *IEEE Trans. Commun.*, vol. 54, no. 1, pp. 23–28, Jan. 2006.