

# Adaptive Channel Allocation for Enabling Target SINR Achievability in Power-controlled Wireless Networks

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**Abstract**—This paper offers a new insight to the fundamental problem of efficient admission control in arbitrary power-controlled wireless networks with an unknown call arrival distribution. Active transmitter-receiver pairs are assumed to (i) communicate simultaneously over shared channels, (ii) define target signal-to-interference and noise ratios (SINRs) by nonlinear functions of channel interference, and (iii) use adaptive power control to maintain the actual SINR at the target level in response to interference variations. Unlike other studies, in this study, power control with limited dynamic range and both the discrete-time and the continuous-time dynamics is explicitly considered, as well as the effects of stochastic radio propagation phenomena. Without relying on *a priori* assumptions, we first define sufficient conditions for a channel allocation mechanism to ensure the SINR constraints in cooperation with the deployed power control mechanism. We use the concept of Lyapunov stability as a cross-layer optimization criterion. Subsequently, we focus on the widely assumed case of SINR targets being defined by linear functions of interference, and show that such targets can be achieved if

$$h_{ii} > |A_i| \sum_{j \neq i} h_{ij} \quad \forall i,$$

where  $h_{ij}$  is the channel gain between the transmitter of link  $j$  and the receiver of link  $i$ , and  $A_i$  is the slope of the linear definition of the target SINR. This knowledge allows us to propose a simple distributed algorithm for implementing an admission control mechanism that (i) uses interference and pilot signal measurements as its only decision-making input, and (ii) allows links to adaptively adjust the SINR targets within the system stability bounds. This mechanism is shown to outperform the carrier sensing approach (CSMA/CA) for admission control.

**Index Terms**—Admission control, power control, SINR achievability, distributed cross-layer optimization, Lyapunov, stability

## I. INTRODUCTION

**I**N this paper, we discuss the problem of efficient admission control in arbitrary power-controlled wireless networks with an unknown call arrival distribution. We consider all

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active network transmissions to be accommodated simultaneously in shared interference-limited communication channels. A transmission is considered to be successful if the signal-to-interference-plus-noise ratio (SINR) at the receiver is higher than a threshold value. Each transmitter-receiver pair (link) is given the freedom to define its own SINR target. Active links then periodically update their transmit powers to compensate for the fluctuations in channel gains and co-channel interference and maintain the actual SINR at the required value.

We consider the SINR targets and the network topology as the initial conditions to achieve our objectives of (i) maximizing the number of active transmissions with the achieved SINR targets, (ii) minimizing the required transmit powers, and (iii) providing feedback on achievable SINR targets for readmission purposes or target SINR-variation purposes. The problem of combined power and admission control in shared channels, under the SINR constraints, is common to both the centralized systems (e.g. the TDD/FDD resource block sharing in the multi-user MIMO/collaborative MIMO uplink mode of the cellular 4G LTE standard [1]) and the distributed systems (sensor, ad hoc, cognitive networks [2]). This problem also occurs as a subproblem in many other areas of wireless networking, often as a result of imperfect orthogonalization of time/frequency/code resources in system implementations.

Although the considered SINR model does not reflect the entire complexity of modern wireless systems, it captures an important aspect of wireless communications, i.e. the actual SINR at the receiver determines the achievable data rate of a link transmission, and it also plays a key role in call initialization, synchronization, equalization, (de)coding etc.. In other words, the quality of service (QoS) of each link depends on the QoS of other links because of the phenomenon of interlink interference in the shared channels.

We study the continuous aspects (i.e. the power allocation dynamics) as well as the discrete aspects (i.e. admission events) of the problem to provide optimized solutions. First, without relying on limiting *a priori* assumptions or technological restrictions as typically adopted in previous studies, Sections IV and VI specify on a theoretical level a set of matched cross-layer optimized algorithms; these algorithms, on the data link (DL) layer, assign active links to available channels such that each channel accommodates transmissions only for those links whose generally nonlinear SINR requirements are mutually compatible and can be satisfied by the power allocation algorithm of the physical (PHY) layer.

Lyapunov stability is used as an optimization criterion for matching the PHY and DL layers. The PHY power control

mechanism is assumed to have continuous-time dynamics and a predefined restricted range for transmit power allocation. The former characteristic assures a good performance in non-ergodic channels, while the latter one models the limitations of the dynamic allocation range by end-stage amplifiers or the requirements of connectivity by the network (NTW) layer.

Subsequently, we apply our framework to the common case of the SINR targets that are defined by linear mappings of co-channel interference to transmit power. We show that, in such a case, the SINR constraint can be satisfied if  $h_{ii} > |A_i| \sum_{j \neq i} h_{ij} \forall i$ , where  $h_{ij}$  is the channel gain between the transmitter of link  $j$  and the receiver of link  $i$ .  $A_i$  is the slope of the linear SINR target definition of link  $i$ .

This knowledge allows us to propose a simple distributed algorithm for implementing an admission control mechanism that (i) uses interference and pilot signal measurements as its only decision-making input, and (ii) allows links to adaptively adjust their SINR targets within the system stability bounds for (re)admission purposes or data rate variation purposes; this algorithm is described in Sections VI and VII. Besides the low complexity and instantaneous decision-making, the proposed algorithm does not require interlink cooperation or data exchange, and it can be employed irrespective of whether the transmit power updates on the PHY layer have discrete-time or continuous-time dynamics.

Numerical simulations, provided in Section VIII, verify the expected ability of the proposed admission control mechanism to non-invasively determine the achievability of a target SINR in a shared channel. The performance of the admission control mechanism is compared with that of the equally complex and widely deployed CSMA/CA approach [3]; results of this comparison show that the proposed admission control mechanism is superior to the widely deployed CSMA/CA approach in terms of its performance. The conclusions of this study are summarized in Section IX.

## II. SUMMARY OF RELATED WORK

The initial studies [4], [5] on the problem of combined power and admission control in shared channels under given SINR constraints *a priori* assume the achievability of the constraints, i.e. the existence and reachability of optimum transmit powers for their satisfaction. The studies [6]–[8] avoid such assumptions; the achievability of the target SINRs is tested using sequential channel probing and iterative data processing. Yet, additional interference and protracted decision making renders such schemes unsuitable for networks that operate on tight energetic budgets or require low call admission delay.

In the case of SINR targets that are defined by nonlinear mappings of transmit power to co-channel interference, the existence of equilibrium transmit powers and their reachability was proved for a class of so-called standard functions [9], [10]. The recent game-theoretical models of power control with resource allocation protocols based on maximization of selected user-satisfaction functions [11]–[14] also fall into this category of nonlinear system models.

The game-theoretical approach allows us to better address certain resource management issues such as fairness; however, the original issue of achieving the predefined SINR targets is

suppressed. Moreover, achieving fairness often implies performance degradation [15]. With regard to the system model universality, studies using game-theoretical approach typically assume application-specific system setups (e.g. CDMA systems as in [12]–[14]) and/or special properties for the employed best-response and utility functions (e.g. positivity, monotonicity, scalability for best-response functions, or supermodularity of utilities as in [16]) in order to ensure that the equilibrium states can be achieved.

On a more general level, all the above studies consider only greedy algorithms with discrete-time dynamics for implementing power control mechanisms. This feature leads to a high volatility of the power allocation dynamics under stochastic radio propagation conditions [10], [17]. Typically, the issues related to transmit power truncations and dynamic channel allocation are not discussed.

## III. SYSTEM MODEL

Assume a wireless network with multiple channels for accommodating the transmissions of multiple mobile users that are distributed arbitrarily over the network area. Channels are given by disjoint subspaces of the time/frequency/code space, and each of them can be shared by several concurrently active links. With no loss of generality, we focus on the case of a single channel shared by  $N$  active links, transmitting continuous sequences of data packets, and we use the notion of signal-to-interference ratio (SIR) instead of SINR since the additive noise is negligible in interference-limited channels.

A transmission is considered to be successful if the SIR at the receiver (RX) is higher than a threshold value. The level of the perceived co-channel interference necessarily varies because of radio propagation phenomena, user mobility and call arrivals/terminations. Therefore, the transmitter (TX) of each active link  $i \in \mathcal{S} = \{1, \dots, N\}$  must periodically update its transmit power  $\sigma_i$  in order to compensate for such fluctuations and maintain its actual SIR level at a desired target level. Let us assume that the power control algorithm of link  $i$  maintains a target  $SIR_i = \beta_i(I_i)/I_i$  at the receiver, where  $\beta_i(I_i) \geq 0$  is an arbitrary time-variant function of co-channel interference  $I_i$  at RX <sub>$i$</sub> . The transmit data rate of link  $i$  is determined using a rate assignment function  $R_i(SIR_i)$ .

The feedback from the RX <sub>$i$</sub>  to the TX <sub>$i$</sub>  (e.g. on the information  $I_i$ ) is assumed to be ideal in order to enable the transmit power updates. Let us assume that all the links perform their updates with a period  $T$  at time instances  $t_i^k$ , which can be generally asynchronous with the timing of other links  $j$ , i.e.  $t_i^k \neq t_j^k$  for an integer  $k$  and any  $i, j \in \mathcal{S}$  and  $j \neq i$ . The period  $T$  is assumed to be sufficiently small to allow links to keep track of network changes. This assumption corresponds to assuming that the mean value  $h_{ij}$  of the time-varying channel gain  $H_{ij}$  between the TX of link  $j$  and the RX of link  $i$  remains quasi-constant within  $T$ . The mean gain  $h_{ij}$  is proportional to the path loss because of physical energy dissipation, large-scale shadowing and, for simplicity, also the spreading/processing gain of CDMA transmissions. The stochastic channel gain function  $H_{ij}(h_{ij})$ , with mean value  $h_{ij}$ , then models all radio wave propagation phenomena including multi-path fading superposed over path loss and shadowing.

In accordance with the standard Open System Interconnection (ISO) model, we suppose that the system architecture, i.e. the implementation of each link, is organized into the following three layers: (i) the PHY layer for carrying out physical transmit power updates, (ii) the DL layer for controlling the link access to the shared channel, and (iii) the NTW layer for data routing based on achieved network connectivity. In this context, the particular manner in which  $\beta_i$  maps the possible values of  $I_i$  into a range  $\Sigma_i$  of acceptable transmit powers  $\sigma_i \in \Sigma_i$  can be chosen individually by each link  $i$  with respect to the configuration of its PHY, DL, and NTW layers. Hence,  $\beta_i$  is discussed in a general manner. The value set  $\Sigma_i$  of  $\beta_i$  is assumed to be finite because of the limitations of the end-stage amplifiers, and also possibly because of the connectivity requirements of the NTW layer. More particularly,  $\Sigma_i = [\sigma_i^{\min}, \sigma_i^{\max}]$  for some  $0 \leq \sigma_i^{\min} < \sigma_i^{\max}$ .

To simplify the used mathematical notation, the set  $\mathcal{S} \setminus \{i\}$  of links  $j$  other than link  $i$  is denoted by  $-i$  and boldface notation for vectors and matrices is introduced. Symbol  $\boldsymbol{\sigma}$  denotes the columnwise-oriented vector of the power values for all  $N$  network links at time  $t$ , i.e.  $\sigma_i$  for all  $i \in \mathcal{S}$ . Analogically,  $\boldsymbol{\sigma}_{-i}$ ,  $\mathbf{h}_{-i}$ , and  $\mathbf{H}_{-i}$  are columnwise-oriented vectors composed of  $N - 1$  elements  $\sigma_j$ ,  $h_{ij}$ , and  $H_{ij}$  for all  $j \in -i$ , respectively. A function denoted by a boldface symbol refers to the Cartesian product of functions, which are denoted by the same symbol in the plain font. For example,  $\boldsymbol{\beta}$  is the Cartesian product of  $\beta_i \forall i$ . The dependencies of  $\sigma_i(t)$ ,  $H_{ij}(h_{ij}(t), t)$ ,  $\beta_i(\mathbf{H}_{-i}(t)^T \boldsymbol{\sigma}_{-i}(t), t)$ , etc. on their arguments are stated only if necessary, otherwise a purely symbolic notation, i.e.  $\sigma_i$ ,  $H_{ij}$ , and  $\beta_i$ , is preferred for clarity.

#### IV. PHY LAYER DESIGN

On the basis of the assumption that the SIR constraints in the shared channel can be satisfied, the above model will be now used for discussing the manner in which optimum TX powers can be distributively and asynchronously allocated on the PHY layer, in order to satisfy the SIR constraints. The existence and reachability of such optimum powers will be assured by admission control actions of a dedicated DL layer, which will be proposed subsequently based on the PHY prerequisites.

An optimally balanced solution to the trade-off between allocating non-zero transmit powers and minimizing the undesirable interference in the shared channel involves the allocation of a set of optimum transmit powers,  $\hat{\boldsymbol{\sigma}}$ , such that

$$h_{ii}\hat{\sigma}_i = \beta_i(\mathbf{h}_{-i}^T \hat{\boldsymbol{\sigma}}_{-i}) \quad \forall i \in \mathcal{S}. \quad (1)$$

Clearly, the search for  $\hat{\boldsymbol{\sigma}}$  corresponds to finding a zero root of the function  $f_i = \frac{1}{h_{ii}}\beta_i(\mathbf{h}_{-i}^T \boldsymbol{\sigma}_{-i}) - \sigma_i$ , i.e. solving

$$\frac{1}{h_{ii}}\beta_i(\mathbf{h}_{-i}^T \boldsymbol{\sigma}_{-i}) - \sigma_i = 0 \quad \forall i \in \mathcal{S}. \quad (2)$$

If the optimum powers  $\hat{\boldsymbol{\sigma}}$  are allocated by the PHY layer, then each link is perfectly adapted with its  $\sigma_i$  to the mutual interference under the given system configuration represented by the corresponding  $\beta_i$ . Then, none of the concurrent links has an incentive to unilaterally deviate therefrom, i.e. change its transmit power, while other active links keep their transmit

powers unchanged. Evidently, if a new link becomes active in the shared channel or conversely terminates its transmission, a new equilibrium state has to be found to dynamically reflect the new network configuration and interference conditions.

In terms of game theory, vector  $\hat{\boldsymbol{\sigma}}$  represents a Nash equilibrium of a non-cooperative power control game in strategic form defined by the three elements  $\{\mathcal{S}, \sigma_i, \beta_i\}$ . It should be noted that the corresponding equilibrium data rates  $\hat{\rho}_i$  can be uniquely determined as  $\hat{\rho}_i = R_i(\text{SIR}_i) = R_i(h_{ii}\beta_i(\mathbf{h}_{-i}^T \hat{\boldsymbol{\sigma}}_{-i})/\mathbf{h}_{-i}^T \hat{\boldsymbol{\sigma}}_{-i})$ . Thus, reaching a network-wide equilibrium in the  $\boldsymbol{\beta}$ -driven power control game also implies the stabilization of the rate adjustment process, irrespective of whether link  $i$  subordinates its choice of  $\beta_i$  to its primary demands on  $R_i$  (e.g. a car user with good energetic supplies) or whether  $R_i$  is derived on the basis of the power restrictions of link  $i$  (e.g. a power-concerned user).

Let us assume now that the shared channel is occupied only by those links whose SIR constraints (i.e.  $\boldsymbol{\beta}$ ) admit non-negative optimum transmit powers  $\hat{\boldsymbol{\sigma}}$  for their satisfaction. Then, the search for  $\hat{\boldsymbol{\sigma}}$  can be carried out on the PHY layer by using distributive iterative algorithms with either discrete-time dynamics or continuous-time dynamics.

#### A. Transmit Power Allocation with Discrete-time Dynamics

The algorithm with discrete-time dynamics is defined in a straightforward manner using (1) as

$$\sigma_i(t_i^k + T) = \begin{cases} h_{ii}(t_i^k)^{-1} \beta_i(\mathbf{H}_{-i}(t_i^k)^T \boldsymbol{\sigma}_{-i}(t_i^k), t_i^k) & \text{if this value} \in \Sigma_i, \\ \sigma_i^* & \text{otherwise.} \end{cases} \quad (3)$$

The iterative updates for  $\sigma_i$  are based directly on  $\beta_i$ , whereby the interference data  $\mathbf{H}_{-i}^T \boldsymbol{\sigma}_{-i}$  represent the only input. All the active links start with an initial TX power  $\sigma_i(t_i^0)$ . More specifically, the transmit power  $\sigma_i$  is updated at time  $t_i^k + T$  to an estimate  $h_{ii}(t_i^k)^{-1} \beta_i(\mathbf{H}_{-i}(t_i^k)^T \boldsymbol{\sigma}_{-i}(t_i^k), t_i^k)$  of the equilibrium component  $\hat{\sigma}_i$ . This estimate must fit into the predefined profile  $\Sigma_i$ , otherwise  $\sigma_i$  is reset to a predetermined value  $\sigma_i^* \in \Sigma_i$ , from which the iterative search for  $\hat{\sigma}_i$  is restarted.

To keep the definition of (3) coherent with that of the next algorithm (4), it is essential that if link  $i$  resets its power to  $\sigma_i^*$  at  $t_i^k$ , the other links  $j \in -i$  join it by collectively switching to  $\sigma_j^*$  at  $t_j^k + T$  too. We denote such a network-wide event as *total reset* of the network PHY layer. The actual redundancy of this requirement for both algorithms, i.e. (3) and (4), will be discussed in Section V and replaced by more natural *partial* (individual) resets in Section VI.

The above implementation of the PHY layer can be effectively used in communication channels with ergodic gains (e.g. channels with fast fading), in which the short-time-scale average of  $H_{ij}$  is equal to the overall average. Then, elementary averaging of input interference data  $\mathbf{H}_{-i}^T \boldsymbol{\sigma}_{-i}$ , collected during each update period  $T$ , is sufficient for eliminating the stochastic nature of channel gains  $H_{ij}$ . Consequently, the search for  $\hat{\boldsymbol{\sigma}}$  on the basis of (3) becomes deterministic.

#### B. Transmit Power Allocation with Continuous-time Dynamics

It is not possible to apply the abovementioned greedy algorithm to channels with non-ergodic gains (e.g. channels

with slow fading), whose short-time-scale average of  $H_{ij}$  is not equal to the overall average  $h_{ij}$ . The reason for this is that large fluctuations in  $\sigma_i$  occur in-between individual updates and degrade the performance of the PHY layer [17] even if averaging of interference data is employed. Nevertheless, in this case, we use stochastic approximation (SA) techniques [18] to distributively implement the search for  $\hat{\sigma}$ .

All the active links start with an initial transmit power  $\sigma_i(t_i^0)$  and directly use the noisy interference data  $\mathbf{H}_{-i}(t_i^k)^T \sigma_{-i}(t_i^k)$  to update their powers on the basis of  $\beta_i$  using the formula

$$\sigma_i(t_i^k + T) = \begin{cases} \sigma_i(t_i^k) + a_i^k f_i(\mathbf{H}_{-i}(t_i^k)^T \sigma_{-i}(t_i^k), t_i^k) & \text{if this value} \in \Sigma_i, \\ \sigma_i^* & \text{otherwise.} \end{cases} \quad (4)$$

The term  $a_i^k$  denotes the algorithmic step size of link  $i$  in time  $t_i^k$ , and  $f_i$  denotes the function  $f_i(\mathbf{H}_{-i}^T \sigma_{-i}) = h_{ii}^{-1} \beta_i(\mathbf{H}_{-i}^T \sigma_{-i}) - \sigma_i$  having the  $i$ -th component  $\hat{\sigma}_i$  of the Nash equilibrium  $\hat{\sigma}$  as its zero root. The total network resets of the power control process must be assumed at this stage (see [18]); however, this assumption will be eliminated by the DL layer design discussed in Sections V and VI.

The dynamics of (4) is continuous, owing to the inherent ability of SA algorithms to average out channel gain fluctuations. From the Arzelà-Ascoli theorem, it is found that the asymptotic properties of (4) are, for large  $k$ , directly related to a standard law of motion, governed for the initial  $\sigma(t^0)$  by  $N$  coupled first-order scalar differential equations given by

$$d\sigma(t)/dt = f(\mathbf{h}(t)^T \sigma(t), t). \quad (5)$$

## V. CROSS-LAYER OPTIMIZATION OF PHY AND DL LAYERS

In this section, we examine the properties of the continuous-time algorithm (4) in order to determine a technical approach for a cross-layer optimized design of the DL layer. Our objective is to (i) assign only those links to the shared channel whose  $\beta$  admits an equilibrium solution  $\hat{\sigma}$  and (ii) support partial (rather than total) resets of the underlying PHY layer.

### A. Relationship Between PHY and DL Layers

Defining the symbol  $\text{diag}(\mathbf{c})$  to represent a diagonal matrix for vector  $\mathbf{c}$ , the conditions for the algorithm (4) with continuous-time dynamics to converge to optimum powers  $\hat{\sigma} \in \Sigma$  can be stated based on [18], [19]:

- I) links iterate recursions (4) with step size  $a_i^k$  such that  $a_i^k > 0$ ;  $a_i^k \xrightarrow{k \rightarrow \infty} 0$ ;  $\sum_{k=0}^{\infty} a_i^k = \infty$ , and  $0 < c_i^{\min} \leq \liminf_{k \rightarrow \infty} a_i^k / a_{j \neq i}^k \leq \limsup_{k \rightarrow \infty} a_i^k / a_{j \neq i}^k \leq c_i^{\max}$  a.s. for some  $c_i^{\min}, c_i^{\max} > 0$ ,
- II) the stochastic nature of channel gains  $H_{ij}$  matches the noise condition of [18], [20],<sup>1</sup>
- III)  $\beta$  is globally Lipschitz in  $\mathbf{h}_{-i}^T \sigma_{-i}$  and piecewise continuous in  $t$ ,
- IV) there exists a twice continuously differentiable scalar function  $v : \Sigma \rightarrow \mathbb{R}^1$  such that  $\sup_{\mathbf{c}_i \in [c_i^{\min}, c_i^{\max}]} \left[ \mathbf{f}^T \text{diag}(1, c_2, \dots, c_N) \text{grad } v \right] < 0$  for all  $\sigma \in \Sigma \setminus \{\hat{\sigma}\}$ , and

<sup>1</sup>See [21] for equivalent expressions of the noise condition.

V) it holds that  $v(\sigma^*) < \inf_{\sigma: \sigma_i = \max[\Sigma_i], \sigma_j \leq \max[\Sigma_{j \neq i}]} v(\sigma)$ .

Under these conditions, algorithm (4) converges even if  $\beta$  varies in time because of changes in the link layer configuration, and  $\beta$  admits multiple equilibria  $\hat{\sigma}$ .

The first three Conditions, i.e. I, II and III, do not provide any useful guidelines for the design of the DL layer. Moreover, they can always be taken for granted - Condition I concerns only the setting of  $a_i^k$ . The noise Condition II restricts, roughly speaking, the variations in  $H_{ij}$  to finite values. The globally Lipschitz property of Condition III holds if  $\beta$  has limited first derivatives, which can be assumed in practical applications.

On the other hand, the other two Conditions, i.e. IV and V, represent an instrumental tool for constructing a DL layer, which can ensure that the PHY layer allocates an optimum solution  $\hat{\sigma}$  in the shared channel, using (4). In particular, from Condition IV, we observe that if the actions of the DL layer can assure the existence of a scalar twice continuously differentiable Lyapunov function  $v$  such that  $\mathbf{f}^T \text{grad } v < 0$ , the existence of  $\hat{\sigma}$  and the convergence of network-wide physical updates (4) towards  $\hat{\sigma}$  can be established.

Moreover, such a design cross-layer optimizes the PHY and DL layers under the notion of overall network stability [22]. The advantage of such a stability-emphasizing approach is that we can allow links to adaptively adjust their  $\beta_i$  under the constraint of the sole *existence* of the auxiliary function  $v$ . Theoretically, this supports the implementation of an adaptive DL layer, because  $\beta$  can be continuously varied without affecting the convergence (stability) of the PHY layer.

Algorithm (4) converges on the whole to  $\hat{\sigma}$ , even if *temporary* instabilities, in terms of Conditions IV and V, arise. Nevertheless, such instabilities should be avoided to promote the energetic efficiency of ongoing transmissions.

Our observations on the design of the DL layer concern the case of the PHY layer being implemented by the continuous-time algorithm (4), but can be also readily extended to the discrete-time counterpart (3) if  $\beta$  is a linear (or linearized) function. In this context, Section VI shows in more detail the duality of the continuous-time and discrete-time searches for  $\hat{\sigma}$ , stemming from the existence of bilinear transformations and their property of preserving Lyapunov functions  $v$ .

### B. Relationship Between PHY and NTW Layers

Both the PHY algorithms, (3) and (4), are defined such that they confine their search for optimum transmit powers  $\hat{\sigma}$  to the power range  $\Sigma$ , whereby the setting of  $\Sigma$  can be controlled by the NTW layer to ensure a certain network connectivity for the employed routing protocol.

For practical reasons, we required above link  $i$  to reset its power to  $\sigma_i^*$  anytime its estimate  $\sigma_i(t_i^k + T)$  of the equilibrium component  $\hat{\sigma}_i$  goes beyond the permissible range  $\Sigma$ . Moreover, all the other links  $-i$  sharing the same channel must join link  $i$  in a network-wide total reset of the PHY layer by compulsorily switching to  $\sigma_{-i}^*$  in their next power updates. However, if the DL layer satisfies the Condition IV by establishing the existence of a *positive-definite* function  $v > 0$  instead of the originally required non-negative  $v$ , the necessity of coordination among network links, for handling such total power control resets on the PHY layer, can be eliminated.

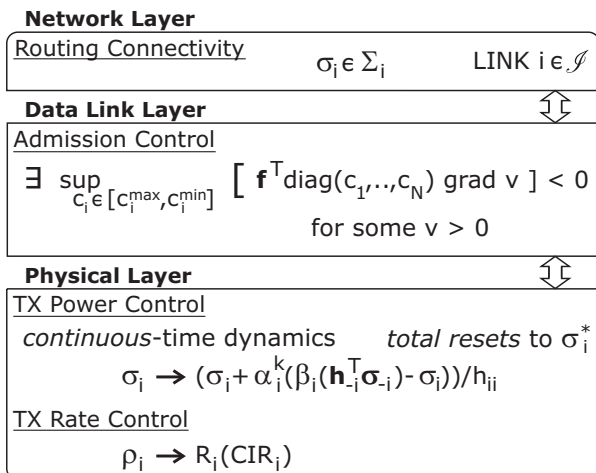


Fig. 1. Proposed cross-layer optimized implementation of link  $i \in \mathcal{S}$  for enabling target SIR achievability in a shared channel under nonlinear  $\beta$ .

Instead, one can implement more practical partial resets in which links  $-i$  ignore the fact as to link  $i$  has reset its power to  $\sigma_i^*$  or not.

To prove this, observe that if the DL layer can satisfy Condition IV by finding a positive-definite  $v$ , then Condition V holds automatically for *any*  $\sigma^* \in \Sigma$ . As such, it also holds for  $\sigma^*$  defined by a combination of components  $\sigma_i^*$  and  $\sigma_j(t_i^k)$  for some  $i \in \mathcal{S}$  and  $\forall j \in -i$ . Hence, partial resets of the power control iterations are possible on the network-wide level.

## VI. DL LAYER DESIGN

### A. Distributed Setup of Stable Networks with Nonlinear $\beta$

Fig. 1 schematically summarizes our previous observations in the form of an adaptive architecture for implementing link  $i$  in wireless networks. The PHY layer uses algorithm (4) with partial resets for allocating optimum transmit powers  $\hat{\sigma}$  within  $\Sigma$ . The definition of  $\Sigma$  can be influenced by the NTW layer. Simultaneously, the PHY layer carries out rate control on the basis of  $\mathbf{R}$ . The DL layer activates individual network links such that the shared channel accommodates transmissions only for those links whose underlying PHY layer requirements, represented by generally nonlinear  $\beta_i$ , collectively ensure the existence of some  $v > 0$ , matching the Condition IV.

The nature of the PHY layer algorithm (4) is distributed by definition. However, the general form of the design of the DL layer does not clearly imply that the DL layer can be implemented in a distributed manner. The next section shows that distributed implementation of the DL layer is possible in case of systems characterized by a linear or linearized  $\beta$ .

### B. Distributed Setup of Stable Networks with Linear $\beta$

Wireless networks whose links have interference-invariant SIR targets can be formalized by  $\beta$ , having linear components with a positive slope. This is observed in the case of current cellular systems. In systems in which the definition of  $\beta$  contains linear components with a negative slope, the link SIR preferences are inversely proportional to the interference. Such systems are then characterized by high-speed high-power transmissions in the case of low co-channel interference, and

in the case of high co-channel interference, they are characterized by low-speed low-power transmissions. The definition of  $\beta$  also incorporates a *receiver*-based admission control of CSMA type (carrier sense multiple access), because if the interference exceeds the level at which  $\beta(\mathbf{h}_{-i}^T \sigma_{-i}) = 0$ , the link becomes passive and waits for better interference conditions.

From a formal point of view, we assume from now on that  $\beta$  is linear with components in the form  $\beta_i(\mathbf{H}_{-i}^T \sigma_{-i}) = B_i + A_i \mathbf{H}_{-i}^T \sigma_{-i}$  for possibly time-variant  $B_i, A_i \in \mathbb{R}^1$ . Further, we also assume first-order differentiability when approximating a nonlinear  $\beta_i$  by a linear one.

The following theorem defines a condition based on which the DL layer can test whether links sharing the same channel have mutually compatible CIR targets in the sense of admitting the existence of an equilibrium  $\hat{\sigma}$ .

**Theorem 1 (Existence of Equilibrium Transmit Powers  $\hat{\sigma}$ ):** Consider a wireless network according to the system model, in which links express their target SIRs by a linear  $\beta_i(\mathbf{H}_{-i}^T \sigma_{-i}) = B_i + A_i \mathbf{H}_{-i}^T \sigma_{-i}$  with time-variant  $B_i, A_i \in \mathbb{R}^1$ . Assume that the DL layer admits links into the shared channel such that there exist  $N$  constants  $d_i > 0$  satisfying  $d_i > \sum_{j \neq i} d_j |A_j h_{ij} / h_{ii}|$  for all  $i \in \mathcal{S}$ . Then, there exists a unique set  $\hat{\sigma} \in \mathbb{R}^N$  of optimum transmit powers satisfying, in the sense of (1), the target SIRs of all links  $\mathcal{S}$ .

See Appendix A for proof.  $\square$

If  $d_i = d_j \forall j \in -i$ , the coefficients  $d_i$  cancel out from the theorem's condition  $d_i > \sum_{j \neq i} d_j |A_j h_{ij} / h_{ii}|$ . In this case, we obtain a set of  $N$  independent conditions  $|A_i| \sum_{j \neq i} h_{ij} / h_{ii} < 1 \forall i$  that can be evaluated *distributively* without the need for any overhead communication or data exchange; each link  $i$  evaluates only the validity of the  $i$ -th condition using its knowledge of  $A_i$  and  $h_{ij} \forall j \neq i$ .

To fully utilize the potential of this observation for a thorough DL layer design, we must prove that testing the condition from *Theorem 1* for  $d_i = d_j \forall j \in -i$  allows not only to verify the existence of  $\hat{\sigma}$  in the shared channel but also the reachability of  $\hat{\sigma}$  by the PHY layer algorithms (3) and (4). The next theorem offers this proof by showing that the distributed evaluation of such a reduced condition ensures the existence of a positive-definite Lyapunov function  $v$  and hence establishes the convergence of algorithms (3) and (4) to equilibrium powers  $\hat{\sigma}$  under partial restarts.

**Theorem 2 (Test of Target SIR Achievability):** Consider a wireless network according to the system model, whose links express their target SIRs by a linear  $\beta_i(\mathbf{H}_{-i}^T \sigma_{-i}) = B_i + A_i \mathbf{H}_{-i}^T \sigma_{-i}$  with time-variant  $B_i, A_i \in \mathbb{R}^1$  and implement the PHY layer using the algorithms (3) or (4). The step size  $\alpha_i^k$  in (4) is defined by Condition I, and also Conditions II and III hold. Assume that the DL layer admits a new link to the shared channel only if all links  $i \in \mathcal{S}$ , comprising the new link and the already herein present links, satisfy the condition

$$h_{ii} > |A_i| \sum_{j \neq i} h_{ij} \quad \forall i. \quad (6)$$

Then, the updates of algorithms (3) or (4) operated under partial resets (i) converge asymptotically to a unique set  $\hat{\sigma} \in \mathbb{R}^N$  of optimum TX powers achieving, in the sense of (1), the target SIRs of all links  $\mathcal{S}$  and (ii) allocate  $\hat{\sigma}$  if  $\hat{\sigma} \in \Sigma$ .

See Appendix B for proof.  $\square$

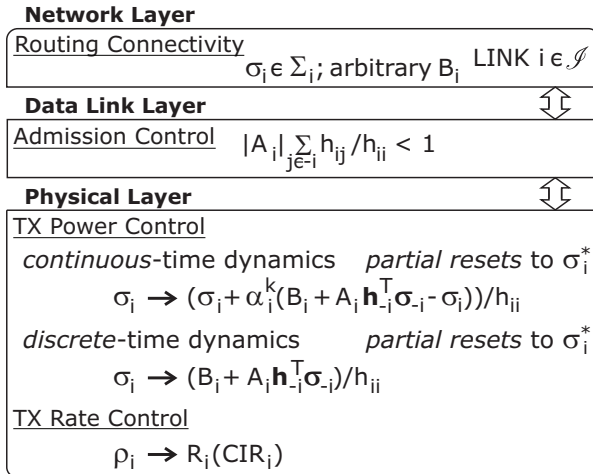


Fig. 2. Proposed cross-layer optimized implementation of link  $i \in \mathcal{S}$  for enabling target SIR achievability in a shared channel under linear(ized)  $\beta$ .

Testing the condition (6) on the DL layer allows links to determine whether accommodating a new transmission with a given SIR target in the shared channel would affect the stability of channel power control dynamics. Analogically, links can set their SIR targets such that Condition (6) holds under the given topology as represented by  $h_{ij}$ . Nevertheless, the dynamics of variations in  $\beta_i$  is practically limited by (i) the ability to keep track of the variations in  $h_{ij}$ , and (ii) the convergence rate of the PHY algorithms to  $\hat{\sigma}$  (their objective is to allocate  $\hat{\sigma}$ , and not just merely converge to it).

*Theorem 2*, in fact, defines a novel link architecture with distributed and asynchronous control; the mutually optimized PHY and DL layers of this architecture allow network links to share a single channel under varying network conditions while always being able to allocate transmit powers that optimally satisfy their target SIRs. Fig. 2 shows the schematic representation of this novel architecture. It should be noted, in this context, that the satisfaction of the terms of *Theorem 2* allows links to eliminate the key assumption of ‘power control feasibility’ in the well-known studies [4], [17].

*Theorem 3* specifies the practical applicability limits of the proposed condition (6) for detecting SIR target achievability on the DL layer. It shows that if one or more links  $i$  fail to satisfy the condition, the PHY layer will not be able to allocate  $\hat{\sigma}$  because of oscillations or divergence.

*Theorem 3 (Applicability Limits of Achievability Test (6)):* Consider a wireless network identical to that considered in *Theorem 2*. Assume that at least one link  $i$  starts to transmit in the shared channel such that  $h_{ii} \leq |A_i| \sum_{j \neq i} h_{ij}$  for at least one  $i$ . Then, the probability that the transmit power allocation dynamics of both the PHY layer algorithms, i.e. (3) and (4), either diverge from  $\hat{\sigma}$  or oscillate around  $\hat{\sigma}$  is non-zero. See Appendix C for proof.  $\square$

From the proof of the theorem, it is found that minor violations of (6) characterized by the term  $|h_{ii} - |A_i| \sum_{j \neq i} h_{ij}|$  being relatively close to zero imply only a low probability of power control divergence. However, the probability of instability increases rapidly with the magnitude of  $|h_{ii} - |A_i| \sum_{j \neq i} h_{ij}|$ . Illustrative quantification of this phenomenon is offered in Section VIII in the context of Fig. 4. Further, it is also found

that *Theorems 1–3* can be equivalently expressed using the concept of fading channel gains  $H_{ij}$  instead of the concept of mean gains  $h_{ij}$ .

## VII. IMPLEMENTATION ISSUES

### A. Implementing DL Layer by Pilot Signal Sensing

Condition (6) allows links to determine the SIR target achievability using the knowledge of channel gains  $h_{ij}$  and parameters  $A_i$ . Parameter  $A_i$  is known to link  $i$  by default. Thus, each link  $i$  is required to only determine the value of  $\sum_{j \neq i}^N \frac{h_{ij}}{h_{ii}}$  in order to be able to test the validity of (6).

As  $\sum_{j \neq i}^N \frac{h_{ij}}{h_{ii}} = \sum_{j \neq i}^N \frac{h_{ij}\sigma}{h_{ii}\sigma} = \frac{I_i}{h_{ii}\sigma}$ ,  $\sum_{j \neq i}^N \frac{h_{ij}}{h_{ii}}$  can be conveniently computed by using a dedicated signalling channel, in which each active transmitter transmits a pilot signal with a constant predetermined power  $\sigma$ . In such a case, the term  $\sum_{j \neq i}^N \frac{h_{ij}}{h_{ii}}$  of link  $i$  equals to the ratio of (i) the value of interference  $I_i$  from other links  $j \neq i$  in the signalling channel, and (ii) the pilot signal strength  $h_{ii}\sigma$  from link  $i$ 's own transmitter TX $_i$ . Contrary to the CSMA-type DL layer, the value of local interference in the data channel is irrelevant. When the value of  $h_{ii}$  is known to link  $i$ , the pilot signal can still be used to determine the term  $\sum_{j \neq i}^N h_{ij}$ , because  $\sum_{j \neq i}^N h_{ij} = \sigma^{-1} \sum_{j \neq i}^N h_{ij}\sigma = \sigma^{-1} I_i$ .

The pilot signal can be a purely analogue electromagnetic wave carrying no modulated information; however, it must be transmitted continuously so that network links can monitor the achievability bound  $\frac{h_{ii}}{\sum_{j \neq i} h_{ij}} > |A_i|$  of their SIR target. In this way, the network links can adapt to network changes caused by mobility or admission events, or they can determine an achievable SIR target before engaging in data transmission.

### B. Role of NTW Layer

The definition of  $\Sigma_i$ , i.e. the setting of parameters  $\sigma_i^{\min}$  and  $\sigma_i^{\max}$ , can be carried out by the NTW layer in order to assure certain network connectivity. The minimum value of  $\sigma_i^{\min}$  and the maximum value of  $\sigma_i^{\max}$  are given by the dynamic range of the end-stage amplifiers. The setting of  $\Sigma_i$  by the NTW layer should be carried out on the basis of the count of partial power control resets on the PHY layer so that the transmit power range  $\Sigma$  contains the *a priori* unknown  $\hat{\sigma}$ . For example,  $\Sigma$  should be expanded if too many successive truncations of the employed PHY layer algorithm occur—a possible indication of the fact that  $\hat{\sigma}_i \notin \Sigma_i$ . However, a certain tolerance in evaluating the frequency of the power control resets is necessary, because excessive powers can be allocated in response to stochastic effects in the shared channel. If further expansions of  $\Sigma_i$  become undesirable in view of the limited energy resources, the transmit power can be simply set to a fixed value, for example to  $\sigma_i^{\max}$ . This value then directly defines a component of a new power control equilibrium.

It should be noted that parameter  $B_i$  does not play any role in the above SIR target achievability test of the DL layer. Hence, the PHY and NTW layers can cooperate in the setting of  $B_i$  in order assure a certain level of network connectivity or control the magnitude of  $\sigma_i$  as  $\hat{\sigma} = -(A - E)^{-1} B$ . Further, from [23], it is also found that the existence test given in *Theorem 1* and the achievability test given in *Theorem 2* can be performed even when the channel gains  $h_{ij}$  vary periodically

(all with the same period). This suggests a certain robustness of the proposed scheme against radio propagation effects with a periodically oscillating nature such as shadowing.

### VIII. NUMERICAL RESULTS

From the results of the numerical simulations, it is concluded that the accuracy of the proposed DL layer algorithm for estimating the achievability of linear SIR targets is better in networks with (i) higher degree of topological randomness, and (ii) higher parameters  $|A_i|$  in the target SIR definition. This conclusion was drawn after comparing the performance of the proposed DL layer algorithm with that of an optimum genie-aided scheme introduced hereafter. Furthermore, it is shown that the proposed DL algorithm substantially outperforms the equally complex and widely deployed CSMA/CA scheme [3].

#### A. Target SIR Achievability Probability in Random Networks

In the simulations, a square network having an area of  $10 \times 10$  km with a single shared channel is used. The average link length is 100 m. Links are implemented on the basis of the distributed layered architecture shown in Fig. 2. All the links chose the same SIR target defined by  $\beta_i = B_i + A_i I_i$  for  $A_i = 5$  or 20. As mentioned in Section VII.2,  $B_i$  is not relevant for DL layer evaluation and is left unspecified. Channel gains  $h_{ij}$  exhibit path loss with exponent 3.5.

By varying the number of active network links  $N$  from 2 to 1500, we generate for each  $N$  exactly 5000 different topologies with links distributed uniformly randomly. Such a setup is denoted as 100% topological randomness.

Analogically, instances of random networks with reduced topological randomness are generated. In these cases, the network area is divided into 100 sections by a regular lattice with 10 rows and 10 columns, and identical square areas are defined around all lattice intersections. The centers of the square areas coincide with the lattice intersections. The centers of individual links are placed uniformly randomly into these square areas. By varying the size of the square areas, the degree of network topological randomness can be changed to a value given basically by the ratio of the square area edge size to the network area edge. For each case of the square area size, the lattice is adjusted such that the square areas fit into the network area. 0% topological randomness corresponds to zero size of the square deviation areas, whereas squares as big as the network area itself yield 100% topological randomness.

For each generated topology, we determine the SIR target achievability using two methods. The first method, i.e. the proposed DL layer scheme, involves the evaluation of the satisfaction of the condition (6). The second method, i.e. an optimally performing, yet genie-aided scheme, involves the evaluation whether the matrix  $(\mathbf{A} - \mathbf{E})$  given in *Theorem 2* is Hurwitz (see proof of the theorem for more details).

Fig. 3 shows the probability of target SIR achievability for both the methods, as a function of active link number  $N$  (data for each  $N$  are averaged over all 5000 generated topologies). The probability is defined as the ratio of the number of topologies with achievable SIR targets in the total of generated topologies.

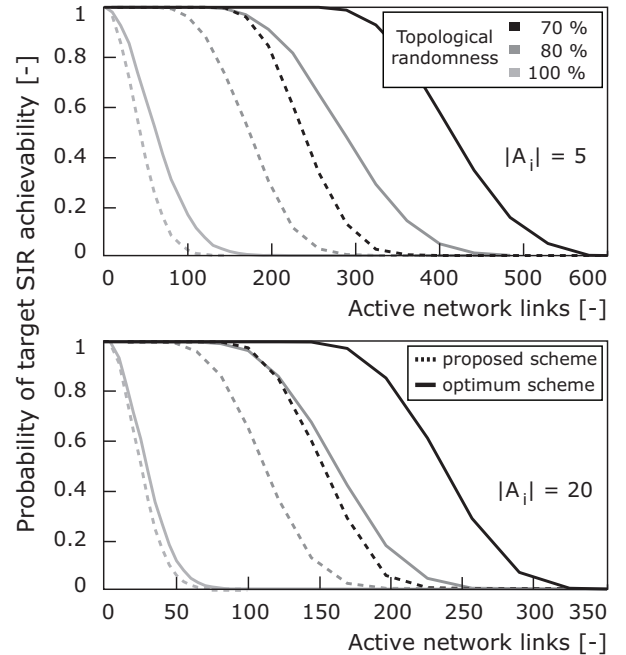


Fig. 3. Probability of target SIR achievability for the proposed and optimum DL layer tests, shown as a function of active link number  $N$  for topological randomness 70%, 80%, and 100% and  $|A_i| = 5$  or 20.

The theoretical probability of target SIR achievability (bold line) naturally decreases with an increase in  $N$ . The proposed test (6) (dashed line) follows this general trend, but with a certain degree of inaccuracy. The performance gap is rather uniform and can be explained using the proof of *Theorem 3*.

As apparent herefrom, the optimum scheme takes advantage of the genie-aided knowledge of the eigenvalues of the matrix  $(\mathbf{A} - \mathbf{E})$ . However, it is a well-known fact that closed-form formulas for expressing the eigenvalues for  $N > 4$  do not exist. To overcome this theoretical limitation, we used Geršgorin disks for estimating the required eigenvalues at the costs of introducing the observed performance gap. Nevertheless, to the best of our knowledge, the proposed test still represents the most general non-invasive and non-iterative method for distributed and cross-layer optimized verification of target SIR achievability in shared channels.

#### B. Accuracy of Detecting Target SIR Achievability

To quantify the accuracy of the proposed DL layer test with respect to the optimum one, we examined in the above topologies the limits  $\Delta A_i^{\min}, \Delta A_i^{\max}$  of the range  $[\Delta A_i^{\min}, \Delta A_i^{\max}]$  such that an SIR target defined by any additive modification of  $A_i$  by an element from this range, i.e.  $A_i + \Delta A_i$  for  $\Delta A_i \in [\Delta A_i^{\min}, \Delta A_i^{\max}]$ , is achievable for all active network links. By default,  $\Delta A_i^{\min}$  and  $\Delta A_i^{\max}$  are set to 0 for topologies with *a priori* unachievable target SIRs.

Fig. 4 shows the average values of  $\Delta A_i^{\min}$  and  $\Delta A_i^{\max}$  as obtained by both the tests and represents them as a function of relative network link density. Relative density is given by the number of active network links normalized by the lowest number of links with a zero probability of target SIR achievability, as shown in Fig. (3).

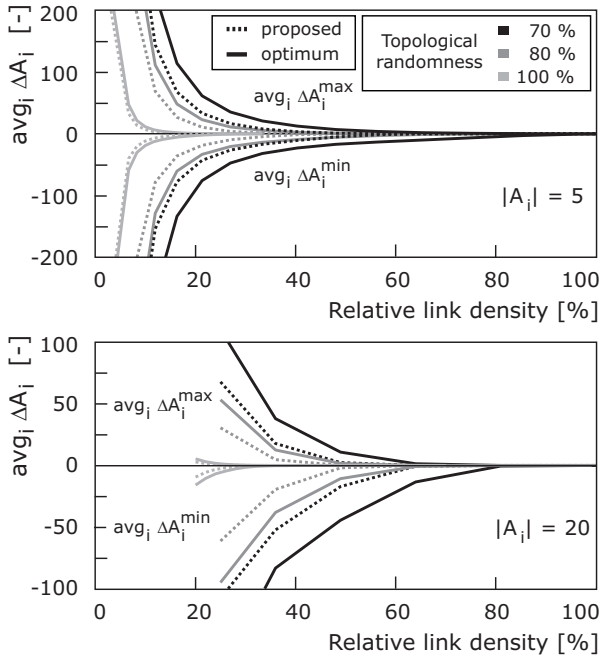


Fig. 4. Average range  $[\Delta A_i^{\min}, \Delta A_i^{\max}]$  for additive modification of  $A_i$  by  $\Delta A_i$  under the achievability constraint as detected by the proposed and optimum DL layer tests, shown as a function of relative network link density for topological randomness 70%, 80%, and 100% and  $|A_i| = 5$  or 20.

We observe that the performance gap between both the tests decreases when the links require higher  $|A_i|$ . Improved accuracy of the proposed test with respect to the optimum scheme can be observed also in the case of higher network densities.

### C. Performance Comparison of the Proposed Scheme with CSMA/CA Approach

We further compare the performance of the proposed scheme with the performance of the equally complex CSMA/CA admission control scheme [3]. This widely deployed CSMA/CA scheme admits link  $i$  to the shared channel if  $\sum_{j \neq i} h_{ij} \sigma^{\text{CS}} = I_i < I^{\text{max}}$ , i.e. if  $\sum_{j \neq i} h_{ij} < I^{\text{max}} / \sigma^{\text{CS}}$ , where  $\sigma^{\text{CS}}$  denotes the predetermined transmit power used under the CSMA/CA scheme, and  $I^{\text{max}}$  is the maximum tolerable co-channel interference. Such a scheme is known as the real CSMA/CA scheme. An ideal CSMA/CA scheme is one in which link  $i$  is admitted such that the interference condition  $\sum_{j \neq i} h_{ij} < I^{\text{max}} / \sigma^{\text{CS}}$  holds for all network links after link  $i$  enters into the network (i.e. the well-known hidden/exposed terminal problem is eliminated).

In particular, we examine the number of links in a random network that each method admits to the shared channel with a predefined SINR target. Such a performance measure is equivalent to the total network capacity. A circular network with a radius of 1000 m and uniformly randomly distributed links is assumed. The link length varies randomly between 100 and 150 m. Channel gains  $h_{ij}$  exhibit path loss with exponent 5. A CSMA/CA transmission is successful when  $\text{SINR} \geq 9.5$  dB. Assuming an additive noise of  $-108$  dB, the proposed scheme uses an equivalent SINR target definition given by  $A_i = 9.5$  dB and  $B_i = 9.5 - 108 = 98.5$  dB.

First, we evaluate the number of successful transmissions under the real and ideal CSMA/CA schemes. The number

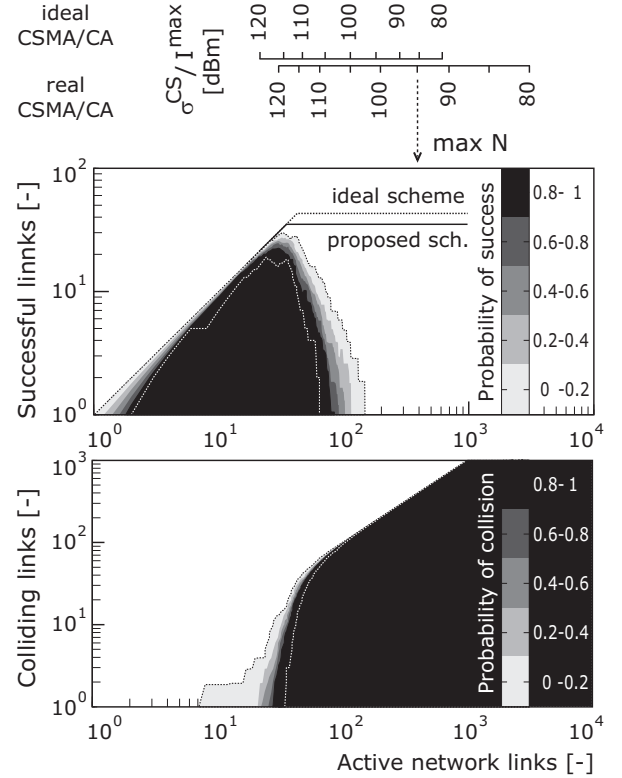


Fig. 5. Number of links with successful or colliding transmission under the CSMA/CA admission control, shown as a function of the total number  $N$  of active network links and compared with the proposed scheme (6) and the optimum scheme. For both the real and the ideal CSMA/CA scheme, maximum  $N$  is indicated for  $\sigma^{\text{CS}} / I^{\text{max}} \in [80, 120]$  dBm.

of so-called colliding transmissions with  $\text{SINR} < 9.5$  dB is measured as well. Fig. 5 shows the results as a function of the active link number  $N$  and compares them with the number of links admitted by the proposed scheme and its optimum counterpart. These two schemes ensure that the SINR targets can be achieved, hence, their number of colliding links is always zero. For both the real and the ideal CSMA/CA schemes, the maximum link number  $N$  obtained at topology saturation (i.e. when  $\sum_{j \neq i} h_{ij} = I^{\text{max}} / \sigma^{\text{CS}}$  for some  $i$ ) is indicated for  $\sigma^{\text{CS}} / I^{\text{max}} \in [80, 120]$  dBm.

As already observed in other studies, the portion of colliding links increases with the link number  $N$ . Only very strict constraints for CSMA/CA decision making with  $\sigma^{\text{CS}} / I^{\text{max}}$  below 100 to 110 dBm solve the collision problem by allowing only relatively few links to share the communication channel. In such a case, the CSMA/CA scheme admits the same number of links, which ensure that the SINR target can be achieved, as those in the case of both the proposed scheme and the optimum scheme. However, if the CSMA/CA admission criterion is relaxed, the CSMA/CA performance degrades significantly, in both the real and the ideal CSMA/CA schemes. For example, already the commonly assumed operation mode with  $\sigma^{\text{CS}} / I^{\text{max}} \sim 90$  dBm causes the number of colliding links to be almost equal to the total link number, i.e. no network link can transmit with the desired SINR. On the other hand, the proposed scheme ensures the SINR of the admitted links. Moreover, the number of admitted links is always greater than that of CSMA/CA, irrespective of the setting of  $\sigma^{\text{CS}}$  and  $I^{\text{max}}$ .

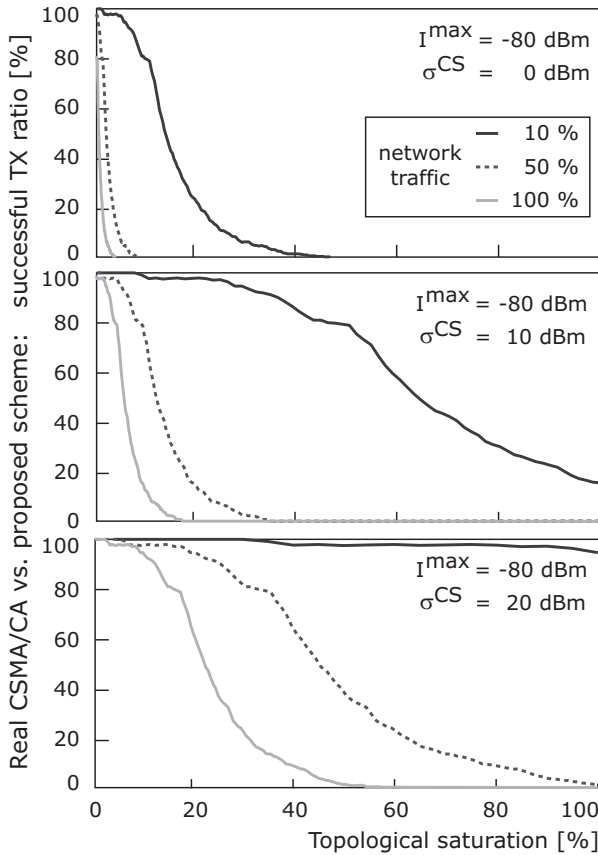


Fig. 6. Ratio of successful transmissions ( $SINR \geq 9.5$  dB) achieved by the real CSMA/CA scheme with respect to the proposed scheme (6), shown as a function of topological saturation. The CSMA/CA scheme assumes TX powers  $\sigma^{CS} = 0, 10, 20$  dBm and interference threshold  $I^{\max} = -80$  dBm. The graphs show the results for 10%, 50%, and 100% of the totally admissible links that are simultaneously active (network traffic).

To illustrate the observed trends in more detail, Fig. 6 shows the ratio of successful transmissions achieved by the real CSMA/CA scheme with respect to those achieved by the proposed scheme. Data are plotted as a function of topological saturation (active link number  $N$  normalized by the maximum  $N$  at topological saturation). The graphs assume scenarios with  $\sigma^{CS} = 0; 10; 20$  dBm for  $I^{\max} = -80$  dBm, and 10%, 50%, and 100% of the totally admissible links that are simultaneously active (various network traffic). Data are averaged over generated topologies.

We observe that as long as the CSMA/CA network saturation and/or network traffic are at very low levels of around 10 to 20%, the performance of the CSMA/CA scheme is the same as that of the proposed scheme. Otherwise, it degrades to a fraction of what could have been achieved by the proposed scheme (graph values approaching 0%). Further simulations verified our observations also for a range of link lengths of 10–150 m, the path loss exponent of 2–6, and  $SINR$  of 0–50.

## IX. CONCLUSION

In this study, we develop novel techniques for ensuring that the target  $SINR$  is achieved by optimized adaptive channel allocation in power-controlled networks with shared channels. Without *a priori* assumptions, we show that assigning network links into shared channels such that their  $SINR$

targets admit the existence of a positive-definite Lyapunov function ensures the convergence of the underlying power control to equilibrium transmit powers that optimally satisfy the  $SINR$  constraints. When the  $SINR$  targets are defined by nonlinear functions of interference, the existence of Lyapunov function(s) represents a sufficient convergence condition for power control algorithms with continuous-time dynamics and partial resets. For linear  $SINR$  targets, the existence condition (i) is necessary and sufficient, (ii) holds for both the discrete-time and the continuous-time power control algorithms under partial resets, and (iii) is fulfilled if  $h_{ii} > |A_i| \sum_{j \neq i} h_{ij}$  for all links  $i$ , where  $h_{ij}$  is the channel gain between  $TX_j$  and  $RX_i$ , and  $A_i$  is the slope of the target  $SINR$  definition. The implementation of practical admission control involves only simple measurements of a pilot signal. The proposed scheme is shown to outperform the equally complex CSMA/CA admission control.

## APPENDIX A PROOF OF *Theorem 1*

Based on the knowledge of  $\beta_i$ , we define  $\mathbf{B}$  to be a column-wise oriented vector with  $B_i/h_{ii}$  in its  $i$ -th row, and  $\mathbf{A}$  to be a matrix with the element in its  $i$ -th row and  $j$ -th column given by  $A_i h_{ij}/h_{ii}$  if  $i \neq j$  and 0 if  $i = j$ . Let  $\mathbf{E}$  be  $N \times N$  unit matrix.

Observe then that the main diagonal of matrix  $(\mathbf{A} - \mathbf{E})$  is negative, because  $\mathbf{A}$  has a zero diagonal and  $\mathbf{E} > 0$ . Moreover, if there exist  $N$  constants  $d_i > 0$  such that  $d_i > \sum_{j \neq i} d_j |A_i h_{ij}/h_{ii}|$  for all  $i \in \mathcal{S}$ , then  $d_i |(A - E)_{ii}| > \sum_{j \neq i} d_j |(A - E)_{ij}|$  for all  $i \in \mathcal{S}$ . Thus, the matrix  $(\mathbf{A} - \mathbf{E})$  is diagonally dominant in the sense of [24], and accordingly  $(\mathbf{A} - \mathbf{E})$  is invertible.

The invertibility of  $(\mathbf{A} - \mathbf{E})$  implies two facts: the inverse matrix  $(\mathbf{A} - \mathbf{E})^{-1}$  exists, and the vector  $\mathbf{s} = \mathbf{0}$  is the only solution of the equation  $(\mathbf{A} - \mathbf{E})\mathbf{s} = \mathbf{0}$ .

Consider now an Euclidean shift of the equation  $(\mathbf{A} - \mathbf{E})\mathbf{s} = \mathbf{0}$  by substituting  $\mathbf{s} = \boldsymbol{\sigma} + (\mathbf{A} - \mathbf{E})^{-1}\mathbf{B}$ . Thereafter, we obtain  $(\mathbf{A} - \mathbf{E})(\boldsymbol{\sigma} + (\mathbf{A} - \mathbf{E})^{-1}\mathbf{B}) = \mathbf{0}$ , which can be simplified into  $\mathbf{B} + (\mathbf{A} - \mathbf{E})\boldsymbol{\sigma} = \mathbf{0}$  and, finally, into  $\mathbf{f} = \mathbf{0}$ . This proves the theorem by showing that there exist a unique vector  $\hat{\boldsymbol{\sigma}} = -(\mathbf{A} - \mathbf{E})^{-1}\mathbf{B}$  solving Eq. (2).

## APPENDIX B PROOF OF *Theorem 2*

This theorem establishes the existence of a unique  $\hat{\boldsymbol{\sigma}}$  based on application of the DL layer test from *Theorem 1* with  $d_i = d_j$  for any  $j \neq i$ . Defining vector  $\mathbf{B}$  and matrices  $\mathbf{A}$  and  $\mathbf{E}$  analogously to the proof of *Theorem 1*, it then follows from this proof that  $\hat{\boldsymbol{\sigma}} = -(\mathbf{A} - \mathbf{E})^{-1}\mathbf{B}$ .

Thus, it only remains to prove that implementing the DL layer based on the achievability test (6) guarantees the convergence of algorithms (3) and (4) under partial resets to  $\hat{\boldsymbol{\sigma}}$ . In accordance with Section V, we do so by relating the achievability test (6) to the existence of  $\nu > 0$  satisfying Condition IV. Note, in this context, that Conditions II and I hold by the theorem definition, whereby linearity and limited slope of  $\beta_i$  in real-life applications satisfy Condition III as  $\|\boldsymbol{\beta}(\mathbf{h}_{-i}^T \boldsymbol{\sigma}_{-i}) - \boldsymbol{\beta}(\mathbf{h}_{-i}^T \boldsymbol{\tau}_{-i})\| = \|\mathbf{A}\boldsymbol{\sigma} - \mathbf{A}\boldsymbol{\tau}\| = \|\mathbf{A}\| \cdot \|\boldsymbol{\sigma} - \boldsymbol{\tau}\|$  in  $\mathbb{R}^N$ .

With no loss of generality, consider substitution of  $s = \sigma - \hat{\sigma}$  into (5), which results in a coordinate shift of  $\hat{\sigma}$  to the origin  $s = \hat{s} = \mathbf{0}$  of  $\mathbb{R}^N$ . Furthermore, suppose a candidate function  $v$  in a polynomial form  $v = \sigma^T \mathbf{V} \sigma$  for some  $N$ -by- $N$  matrix  $\mathbf{V}$ , and define  $\mathbf{C}$  as diagonal matrix  $\mathbf{C} = \text{diag}(1, c_2, \dots, c_N)$ . Then, the derivative of  $v$  along the new shifted trajectories  $\mathbf{f} = \mathbf{B} + (\mathbf{A} - \mathbf{E})(s + \hat{\sigma}) = (\mathbf{A} - \mathbf{E})s$ , scaled component-wise by  $\{1, c_2, \dots, c_N\}$ , is given by  $\dot{v} = \mathbf{f}^T \mathbf{C} \text{grad } v = s^T [\mathbf{V}(\mathbf{C}\mathbf{A} - \mathbf{C}) + (\mathbf{C}\mathbf{A} - \mathbf{C})^T \mathbf{V}] s$ .

It is a known result (e.g. [22]) that there exists a positive-definite symmetric matrix  $\mathbf{V}$  that satisfies

$$\mathbf{W} = -\mathbf{V}(\mathbf{C}\mathbf{A} - \mathbf{C}) + (\mathbf{C}\mathbf{A} - \mathbf{C})^T \mathbf{V}, \quad (7)$$

for some *symmetric positive-definite* matrix  $\mathbf{W}$  if and only if matrix  $(\mathbf{C}\mathbf{A} - \mathbf{C})$  is Hurwitz (i.e. its eigenvalues have negative real parts). So if  $(\mathbf{C}\mathbf{A} - \mathbf{C})$  is Hurwitz for some  $\mathbf{W} > 0$ , then there exists  $\mathbf{V} > 0$  and, consequently,  $v = \sigma^T \mathbf{V} \sigma > 0$  proving the convergence of (4) to  $\hat{\sigma}$  under partial resets.

To show that the above is guaranteed by the DL layer condition (6), we examine Geršgorin's disks [25] of the matrix  $(\mathbf{C}\mathbf{A} - \mathbf{C})$ . Each  $N$ -by- $N$  matrix has  $N$  such disks, and their union identifies the region in the complex plane  $z$  that contains all  $N$  eigenvalues  $\lambda_i$  of  $(\mathbf{C}\mathbf{A} - \mathbf{C})$ . The  $i$ -th disk for the eigenvalue  $\lambda_i$  is given by

$$|z - (\mathbf{C}\mathbf{A} - \mathbf{C})_{ii}| \leq \sum_{j \neq i} |(\mathbf{C}\mathbf{A} - \mathbf{C})_{ij}|, \quad (8)$$

which in our case corresponds to  $|z + c_i| \leq c_i |A_i| \sum_{j \neq i} h_{ij}/h_{ii}$  as  $(\mathbf{C}\mathbf{A} - \mathbf{C})_{ii} = -c_i$  and  $(\mathbf{C}\mathbf{A} - \mathbf{C})_{ij} = c_i A_i h_{ij}/h_{ii}$ . Thus, the Hurwitz nature of  $(\mathbf{C}\mathbf{A} - \mathbf{C})$  is satisfied if and only if  $c_i - c_i |A_i| \sum_{j \neq i} h_{ij}/h_{ii} > 0$ , i.e.  $h_{ii} \geq |A_i| \sum_{j \neq i} h_{ij}$  holds  $\forall i$ . The latter statement corresponds to (6) and, as such, the algorithm (4) converges to  $\hat{\sigma}$ .

It remains to prove the same for the alternative algorithm (3). The continuous-time equation (7) is equivalent in the discrete-time domain to  $\mathbf{A}\mathbf{V}\mathbf{A}^T - \mathbf{V} = -\mathbf{W}$  [22].

Analogously to the case of algorithm (4), a sufficient condition for proving the convergence of algorithm (3) by showing the existence of  $v > 0$  consists in finding a solution  $\mathbf{V} > 0$  for some symmetric positive-definite  $\mathbf{W}$ . This is possible if and only if the eigenvalues  $\lambda_i$  of matrix  $\mathbf{A}$  are such that  $|\lambda_i\{\mathbf{A}\}| < 1$  [26]. We therefore need to prove that the DL layer designed based on (6) assures  $|\lambda_i\{\mathbf{A}\}| < 1$ .

For this purpose, we first bound matrix  $\mathbf{A}$  by an auxiliary matrix  $\mathbf{X}$  such that  $X_{ii} = 0$ ,  $X_{ij} > 0$  and  $|A_{ij}| < X_{ij}$  for  $j \neq i$ . Then, in accordance with Fan's theorem from [25], every eigenvalue  $\lambda_i$  of  $\mathbf{A}$  is located in a disk of the complex plane  $z$  given by

$$|z - A_{ii}| < \lambda_{\max}(\mathbf{X}) - X_{ii}. \quad (9)$$

Since the achievability test (6) implies that  $|A_i| \sum_{j \neq i} h_{ij}/h_{ii} < 1 \forall i$  is satisfied, we can set the bounding matrix  $\mathbf{X}$  to be row (or equivalently column) stochastic, i.e. such that  $\sum_{j \neq i} X_{ij} = 1 \forall i$ . Then by the Perron-Frobenius theorem [25] we obtain  $|\lambda_i(\mathbf{X})| \leq 1$ , whereby equality comes in for exactly one  $i$ .

Recalling that  $A_{ij} = X_{ij} = 0$ , we can rewrite (9) as  $|z| < 1$ , from which follows that  $|\lambda_i\{\mathbf{A}\}| < 1$ . This proves that the achievability test (6) guarantees convergence of the algorithm (3). Its operability under partial resets follows from the fact

that  $v = \sigma^T \mathbf{V} \sigma \rightarrow \infty$  if  $\sigma \rightarrow \infty$  and so is globally stable, i.e. converges for any initial condition  $\sigma^* \in \mathbb{R}^N$  [22].

## APPENDIX C

### PROOF OF Theorem 3

Observe from the proof of Theorem 2 that the PHY layer converges asymptotically to  $\hat{\sigma}$  if and only if  $\text{Re}[\lambda_i(\mathbf{A} - \mathbf{E})] < 0 \forall i$  in the case of power control (4) with continuous-time dynamic, or  $|\lambda_i(\mathbf{A})| < 1 \forall i$  in the case of power control (3) with discrete-time dynamics. However, if roughly speaking an equality holds for at least some links in either of the above inequalities, then the transmit powers  $\sigma$  allocated by the corresponding PHY layer algorithm oscillate around  $\hat{\sigma}$ , whereby divergence follows if an inversion of said inequalities occurs [22], [26].

Using the notion of Geršgorin disks from the proof of Theorem 2, associated with both matrices, we can see that the corresponding eigenvalues,  $\lambda_i(\mathbf{A} - \mathbf{E})$  and  $\lambda_i(\mathbf{A})$  are for all  $i \in \mathcal{I}$ , are located in the union of  $N$  co-centric disks satisfying the properties

- 1)  $|z + 1| \leq r_i$  centered at the point  $[-1, 0]$ ;
- 2)  $|z| \leq r_i$  centered at the origin  $[0, 0]$ ,

with radii equal to  $r_i = |A_i| \sum_{j \neq i} h_{ij}/h_{ii}$ . Thus, it is clear that transmissions violating the DL layer according to Theorem 2, by setting  $r_i \geq |A_i| \sum_{j \neq i} h_{ij}/h_{ii}$ , are the origin of possible oscillations or divergence of the underlying PHY power control.

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