Simple Distributed Algorithm Using Interference Measurements for Optimum Medium Access in Power-Controlled Networks with SINR Constraints

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***Abstract*—This study addresses the general problem of efficient resource management in wireless networks with arbitrary time-varying topologies. We focus our attention on the joint allocation of transmit power and shared communication channels by links (transmitter-receiver pairs), which require a minimum signal-to-interference and noise ratio (SINR) at the receiver for reliable data transfer. The aim is to develop algorithms that allow every network link to determine whether a freely chosen target SINR can be achieved through adaptive power control in a communication channel of interest, using solely simple measurements of local co-channel interference as inputs. The design constraints for derived solutions include (i) a theoretically optimum performance, (ii) minimum complexity in implementation, and (iii) reliable feedback on target SINR achievability to both active and inactive links. Our main contribution is the development of a simple distributed adaptive algorithm, which, in contrast to the related works, solves the optimality/complexity/reliability trade-off without compromising on one or more of these aspects. The algorithm is based on novel and accurate real-time monitoring of the spectral radius of the Foschini-Miljanic matrix. Practical implementations involve measurements of pilot signal strength and/or overall co-channel interference. Our approach achieves the performance upper bounds both in theory and simulations of comparison algorithms that are based on random access, carrier sensing, controlled power-up, or invariant channel probing.[1]**

***Index Terms*—distributed power control and medium access, optimality, dominant eigenvalue, pilot signal, channel probing**

I. Introduction

In this study, we address the need for easy-to-implement yet theoretically optimum resource management in arbitrary wireless networks, which promotes reliable communication among network nodes. In particular, we focus on the allocation of transmit power (battery energy) and shared communication channels. This issue is central to both cellular networks (e.g., the enhanced MIMO modes of the 4G LTE standard [1]) and distributed networks (e.g., ad hoc or sensor networks [2]).

In our system model, described in Section II, many transmitter-receiver pairs (links) are assumed to transmit simultaneously in few communication channels. Active transmitters update their transmit powers with respect to the co-channel interference and noise at the receiver in order to guarantee a minimum signal-to-interference and noise ratio (SINR). Each link can freely select its own preferred target SINR. Within this model, we analyze the problem of joint power control and medium access with given target SINRs under the constraints of *optimality* and *low complexity*.

The "optimality" constraint restricts our attention to solutions in which channel access is granted to all links with achievable target SINRs and refused to others (reuse optimality); at the same time, the transmit powers for maintaining the SINR at each receiver are minimized (energetic optimality).

The "low complexity" constraint primarily refers to the constraint on the complexity of gathering input data for internal decision-making; this constraint must be considered since any communication overhead degrades the performance of wireless networks. More specifically, we impose the condition that network links must make resource allocation decisions solely on the basis of distributed interference measurements. This approach has been successfully applied for developing simple yet largely suboptimal carrier-sensing schemes for resource management (e.g., CSMA/CA medium access mechanisms in the IEEE 802.11 standards [3] or power control with constant SINR constraints [1], [2], [4]). The complexity of internal data processing is of secondary importance, given the low cost and high performance of modern microprocessors; yet, our proposed solutions require only a few arithmetic operations.

Moreover, it is also desirable that all our solutions help to provide an updated feedback on SINR achievability to both active and inactive nodes. Apart from assisting medium access control, such feedback allows network links to individually or collectively adjust their target SINR requirements in order to ensure that the global target SINR is achieved under actual networking conditions. As discussed subsequently, in related studies, the constraints on optimality, complexity, or achievability of feedback have been compromised to simplify the corresponding trade-offs.

In this context, it is well-known that a given set of target SINRs can be optimally achieved *if and only if* the absolute value of the dominant eigenvalue (i.e., the spectral radius) of the Foschini-Miljanic matrix $\boldsymbol{A}$ is less than one [4]. On the basis of this necessary and sufficient condition, we present a novel distributed algorithm in Section IV for computing the above mentioned dominant eigenvalue under varying networking conditions. Despite the locally unknown nature of $\boldsymbol{A}$ and the extreme complexity of standard numerical methods, our algorithm efficiently performs eigenvalue calculation only using simple local interference measurements as input.

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The real-time ability to distributively monitor the value and variations of the dominant eigenvalue of $\boldsymbol{A}$ allows us to formulate an original solution to the resource management problem in wireless networks. The solution has the form of matched algorithms for distributed power control and medium access and satisfies all the abovementioned design constraints. Practical implementations of the algorithm involve measurements of pilot signal strength and/or co-channel interference.

Simulation results of Section VI confirm that our approach helps achieve the performance upper bounds of related algorithms that are based on random access, carrier sensing [3], channel probing [5], [6], or power-up [7]. The conclusion is presented in Section VII.

II. System Model

We assume a standard system model for wireless power-controlled networks [4]–[7] while making one important modification. The target SINR is not assumed to be a non-negative constant; rather, it is defined as a function of co-channel interference, which can differ from link to link. Thus, the matrix $\boldsymbol{A}$ can have negative elements, and the widely applied results of the non-negative matrix theory, such as the Perron-Frobenius theorem, become inapplicable.

The network nodes are distributed arbitrarily over the network area and communicate over freely shared channels. A channel is defined by a combination of time interval(s) with a CDMA spreading code and a frequency band. Without loss of generality, we consider only a single shared channel. The channel gain between the transmitter (TX) of link i and the receiver (RX) of link j will be denoted as h_{ij}.

To maintain the quality and reliability of wireless connections over a period of time, the transmitter of each link i periodically updates its transmit power σ_i. Denoting the additive noise of link i as n_i, we formally assume that the transmit power σ_i of link i is updated in time $k+1$ on the basis of an arbitrary linear function of the receiver interference $I_i(k) = \sum_{\forall j \neq i} h_{ij}\sigma_j(k) + n_i$ in time k, which, for some initial transmit power $\sigma_i(0)$, is given by

$$\sigma_i(k+1) = \frac{b_i + a_i I_i(k)}{h_{ii}} \qquad a_i, b_i \in \Re. \tag{1}$$

Such updates correspond to maintaining a target $\mathrm{SINR}_i = a_i + b_i/I_i$. The channel gains h_{ij} are assumed to be invariant over the update period between time k and $k+1$ but can vary on a larger time scale because of mobility or slow fading.

Note that all links are allowed to freely select (and vary) the values of their two target SINR parameters a_i and b_i. In this manner, they can react to diverse networking conditions and adaptively express their quality-of-service (QoS) requirements.

For example, links with a demand for a constant interference-independent SINR can choose $b_i = 0$ and $a_i > 0$. Previous studies typically restricted their attention to this case to take advantage of non-negative matrix theory. However, energy-consumption concerned nodes can set $b_i > 0$ and $a_i < 0$ to enable bursty transmissions with high SINR and data rate during low co-channel interference. Various settings of $b_i \neq 0$ and $a_i \neq 0$, such as $b_i > 0$ and $a_i > 0$, model power control loops of the cellular LTE standard, in which the TX power is adaptively assigned on the basis of proximity of the RX to the cell edge, number of used resource blocks, PUSCH transport format, etc. [1]. An appropriate choice of a_i and b_i can also guarantee that the received power at the receiver exceeds a threshold level required for synchronization and equalization.

III. Summary of Previous Work

The transmit power allocation dynamics of (1) attains an equilibrium state if $\sigma_i(k) = \sigma_i(k+1) = b_i h_{ii}^{-1} + a_i\left(\sum_{\forall j \neq i} h_{ij}\sigma_j(k) + n_i\right) h_{ii}^{-1}$ $\forall i$. Denoting the vector with the i-th component given by $\sigma_i(k)$ by $\boldsymbol{\sigma}(k)$ and the vector of equilibrium powers by $\hat{\boldsymbol{\sigma}}$, it follows that $\hat{\boldsymbol{\sigma}} = \boldsymbol{B} + \boldsymbol{A}\hat{\boldsymbol{\sigma}}$ or

$$\hat{\boldsymbol{\sigma}} = -(\boldsymbol{A} - \boldsymbol{E})^{-1}\boldsymbol{B}, \tag{2}$$

where the i-th component of vector $\boldsymbol{B}$ is $(b_i + a_i n_i)/h_{ii}$, the element in the i-th row and j-th column of matrix $\boldsymbol{A}$ is given by $a_i h_{ij}/h_{ii}$ if $i \neq j$ and 0 if $i = j$, and $\boldsymbol{E}$ is a unit matrix.

A unique (and Pareto optimum) equilibrium vector $\hat{\boldsymbol{\sigma}}$ from (2) exists and the power control algorithm (1) asymptotically converges to $\hat{\boldsymbol{\sigma}}$, *if and only if* the channel gains h_{ij} and parameters a_i are such that (i) $\boldsymbol{A}$ has a full rank, and (ii)

$$\max_{\forall i} |\lambda_i| \leq 1, \tag{3}$$

where any λ_i, such that $|\lambda_i| = 1$, corresponds to a Jordan cell with a dimension of one [8]. Moreover, if condition (ii) does not hold, the algorithm (1) diverges.

Note that the condition (3) for the existence and reachability of $\hat{\boldsymbol{\sigma}}$ is *necessary* and *sufficient*. In other words, it is theoretically impossible to devise an *optimum* resource management scheme for nodes with given target SINRs without directly or indirectly considering the value of the dominant eigenvalue of $\boldsymbol{A}$. Unfortunately, the computation of matrix eigenvalues is difficult. General closed-form formulas for networks with more than four links do not exist (according to the well-known Abel-Ruffini theorem), and standard numerical methods are extremely complex due to, for example, the assumption of distributed matrix inversion. Moreover, the value of the dominant eigenvalue is different in distinctive channels and varies over time owing to link mobility, medium access events, and variations in channel gains and target SINRs.

To overcome these drawbacks, low-complexity solutions with acceptable suboptimum performance were developed using random access techniques (e.g. ALOHA) or carrier sensing with interference level evaluation (e.g. the popular IEEE 802.11 DC function [3]). However, the performance of such solutions suffers from unpredictable packet collisions.

In other studies, where non-negative interference-invariant target SINRs were typically considered, channel allocation schemes that are based on an approximation of the abovementioned dominant eigenvalue have been proposed. The schemes with the lowest complexity [5], [9], [10] use local interference measurements as input. The use of a series of multiple channel probes with constant power was proposed in [6]. However, inaccurate eigenvalue estimates of [5], [6], [9] cause the herein proposed medium access mechanisms to unnecessarily reject otherwise admissible links. The scheme of [10] theoretically solves this problem, but issues related to the setting of probing powers and the timing of admission control decisions reduce its practical applicability. Both delay and energy dissipation are inherent to another optimum scheme [7], which combines slow power-up with watchdog-type control mechanisms.

Importantly, all the above solutions do not provide links with any feedback on target SINR achievability under mobility or after new medium access events. This further degrades their potential for use in real-life applications.

IV. Distributed Optimum Algorithm For Iterative Channel Monitoring and Medium Access

A. Main Result

We observed in (3) that the knowledge of the dominant eigenvalue of $\boldsymbol{A}$ is essential for any network link to optimally determine whether its target SINR is achievable in a desired channel. In this subsection, we present an efficient technique that allows to use interference measurements for computing the value of the dominant eigenvalue. This result will then be used in the following subsections for implementing the monitoring of target SINR achievability and optimum medium access.

For mathematical purposes, we assume that the algebraic multiplicity of the dominant eigenvalue of $\boldsymbol{A}$ is one. We also assume that, generally speaking, the vectors $\boldsymbol{B}$ and $\boldsymbol{\sigma}(0)$ are not orthogonal to the associated dominant eigenvector of $\boldsymbol{A}$.

Assumption 1 (On $\boldsymbol{A}$ and Its Dominant Eigenvalue λ_1): The eigenvalues of $\boldsymbol{A}$, denoted by λ_i, are indexed decreasingly by the order of magnitude. Then, it is assumed that (i) $\boldsymbol{A}$ is diagonalizable and (ii) the dominant eigenvalue of $\boldsymbol{A}$, denoted as λ_1 is such that $|\lambda_1| > |\lambda_{i\neq 1}|$. □

Assumption 2 (On $\boldsymbol{\sigma}(0)$ and $\boldsymbol{B}$): Denote by $\boldsymbol{x}_i$ the eigenvector of $\boldsymbol{A}$, associated with the eigenvalue λ_i. If $\boldsymbol{B} = \boldsymbol{0}$, then it is assumed that the vector $\boldsymbol{\sigma}(0)$ is not orthogonal to the dominant eigenvector $\boldsymbol{x}_1$ of $\boldsymbol{A}$, associated with λ_1. If $\boldsymbol{B} \neq \boldsymbol{0}$, then it is assumed that the vector $\boldsymbol{\sigma}(0) + \boldsymbol{B}/(\lambda_1 - 1)$ is not orthogonal to $\boldsymbol{x}_1$ for $|\lambda_1| > 1$; the vector $\boldsymbol{B}$ is not orthogonal to $\boldsymbol{x}_i$ for at least one i for $|\lambda_1| < 1$ and $\lambda_1 = -1$; and the vector $\boldsymbol{B}$ is not orthogonal to $\boldsymbol{x}_1$ for $\lambda_1 = 1$. □

Then the following theorem holds:

Theorem 1 (Distributed Monitoring of λ_1 by $\boldsymbol{n}$ Links): Assume that a total of N active links transmit simultaneously over a shared channel while updating their transmit powers based on (1). Define $\boldsymbol{\sigma}_{\boldsymbol{n}}(k)$ as the vector of transmit powers, which are allocated in time k by an arbitrary non-empty subset $\boldsymbol{n}$ of the N active links. For some bounded delay $t \geq 0$, also define the sequence

$$\mathscr{W}_{\boldsymbol{n}}(k,t) = \frac{[\boldsymbol{\sigma}_{\boldsymbol{n}}(k+1+t) - \boldsymbol{B}_{\boldsymbol{n}}]^{\mathrm{T}} \boldsymbol{\sigma}_{\boldsymbol{n}}(k)}{\boldsymbol{\sigma}_{\boldsymbol{n}}(k)^{\mathrm{T}} \boldsymbol{\sigma}_{\boldsymbol{n}}(k)}. \tag{4}$$

Then, the sequence $\mathscr{W}_{\boldsymbol{n}}(k,t)$ converges asymptotically to a limit $W_{\boldsymbol{n}}(t) = \lim_{k\to\infty} \mathscr{W}_{\boldsymbol{n}}(k,t)$ such that, for some real $f \in \Re$,

$$W_{\boldsymbol{n}}(t) = \begin{cases} \lambda_1 f;\ |\lambda_1 f| \leq 1 & \text{for } |\lambda_1| < 1 \text{ and } \boldsymbol{B}_{\boldsymbol{n}} \neq \boldsymbol{0} \\ \lambda_1^{t+1} & \text{otherwise, except for} \\ & \quad \lambda_1 = -1 \text{ and } \boldsymbol{B}_{\boldsymbol{n}} \neq \boldsymbol{0}. \end{cases} \tag{5}$$

If $\lambda_1 = -1$ and $\boldsymbol{B}_{\boldsymbol{n}} \neq \boldsymbol{0}$, the limit $W_{\boldsymbol{n}}(t)$ does not exist. □

Proof: The dynamics of transmit power updates based on (1) can be expressed for $k \geq 0$ as

$$\boldsymbol{\sigma}(k) = \boldsymbol{B} + \boldsymbol{A}\boldsymbol{\sigma}(k-1) = \boldsymbol{A}^k \boldsymbol{\sigma}(0) + \sum_{n=0}^{k-1} \boldsymbol{A}^n \boldsymbol{B}, \tag{6}$$

wherein $\boldsymbol{A}^0$ is a unit matrix.

Assumption 1 assures thanks to its requirement of diagonizable $\boldsymbol{A}$ the existence of a linearly independent set of N eigenvectors $\boldsymbol{x}_i$ of $\boldsymbol{A}$ such that $\boldsymbol{A}\boldsymbol{x}_i = \lambda_i \boldsymbol{x}_i$ for any i. The N eigenvectors $\boldsymbol{x}_i$ can be therefore taken as a basis for expressing the initial power vector $\boldsymbol{\sigma}(0)$ as $\boldsymbol{\sigma}(0) = \sum_{i=1}^N c_i \boldsymbol{x}_i$ for some N constants c_i. Analogically $\boldsymbol{B} = \sum_{i=1}^N d_i \boldsymbol{x}_i$ for some N constants d_i. Then $\boldsymbol{\sigma}(k)$ can be expressed as

$$\boldsymbol{\sigma}(k) = \sum_{i=1}^{N} \left(c_i \lambda_i^k + \sum_{n=0}^{k-1} d_i \lambda_i^n \right) \boldsymbol{x}_i = \sum_{i=1}^{N} g_i(k)\, \boldsymbol{x}_i, \tag{7}$$

where $g_i(k) = \begin{cases} \left(c_i + \frac{d_i}{\lambda_i - 1}\right) \lambda_i^k - \frac{d_i}{\lambda_i - 1} & \lambda_i \neq 1 \\ c_i + d_i k & \lambda_i = 1. \end{cases}$ The definition of $g_i(k)$ is based the fact that the term $\sum_{n=0}^{k-1} \lambda_i(n)$ can be expressed as a sum $\frac{\lambda_i^k - 1}{\lambda_i - 1}$ of a geometrical series with the quotient y_i and initial value 1 if $\lambda_i \neq 1$. Thus,

$$\mathscr{W}_{\boldsymbol{n}}(k,t) = = \frac{(\boldsymbol{\sigma}_{\boldsymbol{n}}(k+t+1) - \boldsymbol{B}_{\boldsymbol{n}})^{\mathrm{T}} \boldsymbol{\sigma}_{\boldsymbol{n}}(k)}{\boldsymbol{\sigma}_{\boldsymbol{n}}(k)^{\mathrm{T}} \boldsymbol{\sigma}_{\boldsymbol{n}}(k)} = \tag{8}$$

$$= \lambda_1 \frac{\sum_{i=1}^N \sum_{l=1}^N \lambda_i g_i(k+t)\, g_l(k)\, \boldsymbol{x}_{i,\boldsymbol{n}}^{\mathrm{T}} \boldsymbol{x}_{l,\boldsymbol{n}}}{\sum_{i=1}^N \sum_{l=1}^N \lambda_1 g_i(k)\, g_l(k)\, \boldsymbol{x}_{i,\boldsymbol{n}}^{\mathrm{T}} \boldsymbol{x}_{l,\boldsymbol{n}}} \tag{9}$$

If $\boldsymbol{B}_{\boldsymbol{n}} = 0$, then $g_i(k) = c_i \lambda_i^k$. Knowing that

$$\lim_{k\to\infty} \frac{g_i(k+t)}{\lambda_1^k} = \begin{cases} c_1 \lambda_1^t & i = 1 \\ 0 & i \neq 1, \end{cases} \tag{10}$$

we obtain, thanks to $\boldsymbol{x}_1^{\mathrm{T}} \boldsymbol{\sigma}(0) \neq 0$ (i.e., $c_1 \neq 0$),

$$W_{\boldsymbol{n}}(t) = \lim_{k\to\infty} \mathscr{W}_{\boldsymbol{n}}(k,t) \frac{\lambda_1^{-2k}}{\lambda_1^{-2k}} = \lambda_1 \frac{\lambda_1 \lambda_1^t c_1^2 \boldsymbol{x}_{1,\boldsymbol{n}}{}^{\mathrm{T}} \boldsymbol{x}_{1,\boldsymbol{n}}}{\lambda_1 c_1^2 \boldsymbol{x}_{1,\boldsymbol{n}}{}^{\mathrm{T}} \boldsymbol{x}_{1,\boldsymbol{n}}} = \lambda_1^{t+1}.$$

The limit exists as we required $\boldsymbol{x}_1^{\mathrm{T}} \boldsymbol{\sigma}(0) \neq 0$, i.e., $c_1 \neq 0$, for $\boldsymbol{B} = \boldsymbol{0}$.

To determine $W_{\boldsymbol{n}}(t) = \lim_{k\to\infty} \mathscr{W}_{\boldsymbol{n}}(k,t)$ for $\boldsymbol{B}_{\boldsymbol{n}} \neq 0$, we distinguish 3 cases (i) $|\lambda_1| > 1$, (ii) $|\lambda_1| < 1$, and (iii) $|\lambda_1| = 1$:

(i) If $|\lambda_1| > 1$, then it holds for a bounded t that

$$\lim_{k\to\infty} \frac{g_i(k+t)}{\lambda_1^k} = \begin{cases} \lambda_1^t \left(c_1 + \frac{d_1}{\lambda_1 - 1}\right) & i = 1 \\ 0 & i \neq 1. \end{cases} \tag{11}$$

Thus, $W_{\boldsymbol{n}}(t) = \lim_{k\to\infty} \mathscr{W}_{\boldsymbol{n}}(k,t) \frac{\lambda_1^{-2k}}{\lambda_1^{-2k}} = \lambda_1 \frac{\lambda_1 \lambda_1^t \left(c_1 + \frac{d_1}{\lambda_1 - 1}\right)^2 \boldsymbol{x}_{1,\boldsymbol{n}}{}^{\mathrm{T}} \boldsymbol{x}_{1,\boldsymbol{n}}}{\lambda_1 \left(c_1 + \frac{d_1}{\lambda_1 - 1}\right)^2 \boldsymbol{x}_{1,\boldsymbol{n}}{}^{\mathrm{T}} \boldsymbol{x}_{1,\boldsymbol{n}}} = \lambda_1^{t+1}$. The limit exists as we required $\boldsymbol{x}_1^{\mathrm{T}}\left(\boldsymbol{\sigma}(0)+\frac{\boldsymbol{B}}{\lambda_1-1}\right)\neq 0$, i.e., $\left(c_1+\frac{d_1}{\lambda_1-1}\right)\neq 0$, for $\boldsymbol{B}\neq\mathbf{0}$.

(ii) If $|\lambda_1|<1$, then $\lim_{k\to\infty} g_i(k+t)=\frac{d_i}{1-\lambda_i}$, thus,

$$W_{\boldsymbol{n}}(t)=\lambda_1\frac{\sum_{i=1}^{N}\lambda_i\sum_{l=1}^{N}\frac{d_i}{1-\lambda_i}\frac{d_l}{1-\lambda_l}{\boldsymbol{x}_{e,\boldsymbol{n}}}^{\mathrm{T}}\boldsymbol{x}_{l,\boldsymbol{n}}}{\sum_{i=1}^{N}\lambda_1\sum_{l=1}^{N}\frac{d_i}{1-\lambda_i}\frac{d_l}{1-\lambda_l}{\boldsymbol{x}_{e,\boldsymbol{n}}}^{\mathrm{T}}\boldsymbol{x}_{l,\boldsymbol{n}}}. \tag{12}$$

The limit exists as we required $\boldsymbol{x}_i^{\mathrm{T}}\frac{\boldsymbol{B}}{\lambda_i-1}\neq 0$, i.e., $\frac{d_i}{\lambda_i-1}\neq 0$, for at least one i. Clearly, $|W_{\boldsymbol{n}}(t)|\leq 1$ with equality holding if $d_i 0$ for all $i\neq 1$.

(iii) For $|\lambda_1|=1$, two subcases must be distinguished. If $\lambda_1=1$, then

$$\lim_{k\to\infty}\frac{g_i(k+t)}{k}=\begin{cases}d_1 & i=1\\ 0 & i\neq 1.\end{cases} \tag{13}$$

Accordingly,

$$W_{\boldsymbol{n}}(t)=\lim_{k\to\infty}\mathscr{W}_{\boldsymbol{n}}(k,t)\frac{k^{-2}}{k^{-2}}=\lambda_1\frac{\lambda_1 d_1^2{\boldsymbol{x}_{1,\boldsymbol{n}}}^{\mathrm{T}}\boldsymbol{x}_{1,\boldsymbol{n}}}{\lambda_1 d_1^2{\boldsymbol{x}_{1,\boldsymbol{n}}}^{\mathrm{T}}\boldsymbol{x}_{1,\boldsymbol{n}}}=\lambda_1.$$

The limit exists as we required $\boldsymbol{x}_1^{\mathrm{T}}\boldsymbol{B}\neq 0$, i.e., $d_1\neq 0$. On the other hand, if $\lambda_1=-1$, the limit $W_{\boldsymbol{n}}(t)$ does not exist due to oscillations of the terms λ_1^{k+t} and λ_1^{k} between 1 and -1 in the sequence $\mathscr{W}_{\boldsymbol{n}}(k,t)$. However, $\mathscr{W}_{\boldsymbol{n}}(k,t)$ can be approximated, for sufficiently large k, to be oscillating around the limit (12).

■

We observe from the theorem that the sequence $\mathscr{W}_{\boldsymbol{n}}(k,t)$ converges for increasing time k to a limit $W_{\boldsymbol{n}}(t)$ whose value is a function of λ_1. Moreover, it holds that $W_{\boldsymbol{n}}(t)=1$ if $\lambda_1=1$, $|W_{\boldsymbol{n}}(t)|>1$ if $|\lambda_1|>1$, and $|W_{\boldsymbol{n}}(t)|<1$ if $|\lambda_1|<1$ (an equality is obtained in the last case if the constant d_i introduced in the proof is 0 for all $i\neq 1$). The limit case in which λ_1 is equal *exactly* to -1 and the limit $W_{\boldsymbol{n}}(t)$ does not exist can be neglected because such a condition is unlikely to occur and exist for a significant time interval under practical conditions.

Hence, by iteratively computing the terms of the sequence $\mathscr{W}_{\boldsymbol{n}}(k,t)$ and comparing their limit $W_{\boldsymbol{n}}(t)$ to the threshold value 1, any arbitrary non-empty set $\boldsymbol{n}$ of active network links can optimally determine, in the sense of (3), whether its target SINRs are achievable ($|W_{\boldsymbol{n}}(t)|<1$) in the desired channel or are not achievable ($|W_{\boldsymbol{n}}(t)|>1$). The terms $\mathscr{W}_{\boldsymbol{n}}(k,t)$ are computed on the basis of (i) the *a priori* known vector $\boldsymbol{B}_{\boldsymbol{n}}$ and (ii) the TX powers $\boldsymbol{\sigma}_{\boldsymbol{n}}$ at time k and $k+1+t$. The latter are computed from the measurements of co-channel interference at times $k-1$ and $k+t$ by using (1). Thus, measurements of local co-channel interference at RX are sufficient for evaluating the target SINR achievability.

In theory, different subsets $\boldsymbol{n}$ of active links generate different sequences $\mathscr{W}_{\boldsymbol{n}}(k,t)$ with different rates of asymptotic convergence. However, the limit $W_{\boldsymbol{n}}(t)$ of all these sequences does not vary and is defined by the unique dominant eigenvalue λ_1 of $\boldsymbol{A}$. The case $\boldsymbol{n}=\{i\}$ is of the greatest interest from a practical viewpoint; in this case, in which each network link i generates the sequence $\mathscr{W}_{\boldsymbol{n}=\{i\}}(k,t)$ independently from others by using only the information of its own TX powers $\sigma_i(k)$ and the parameter b_i.

With respect to the decision-making delay, the condition (3) only requires us to determine whether $|\lambda_1|<1$. In other words, λ_1 does not need to be estimated with an infinitesimal precision that theoretically requires computation of infinite number of terms $\mathscr{W}_{\boldsymbol{n}}(k,t)$ for $k\to\infty$. Numerical simulations in Section VI (Fig. 1) indicate that only between one and four interference measurements (one per power update cycle) are typically sufficient to make consistently correct decisions.

B. Monitoring of Target SINR Achievability

A straightforward application of *Theorem 1* is the monitoring of the achievability of target SINRs by active links. In such a case, each set $\boldsymbol{n}$ of active links computes in each time k the term $\mathscr{W}_{\boldsymbol{n}}(k,t)$ and performs a comparison with the threshold 1. Setting the time shift t between the data samples to zero enables the most up-to-date monitoring.

A fully distributed (individual) monitoring scheme with no data exchange, i.e., a communication overhead is obtained for $|\boldsymbol{n}|=1$. Cellular implementations can take advantage of the central role of base station in each cell and compute $\mathscr{W}_{\boldsymbol{n}}(k,t)$ based on collective data measurements by generally $\boldsymbol{n}$ links with $|\boldsymbol{n}|>1$. It is observed that the more links in $\boldsymbol{n}$, the greater the numerical robustness of the proposed method [11]. It is apparent from *Theorem 1* that the results of collective interference measurements can be disseminated with a varying delay t without affecting the final decision-making accuracy.

Owing to the ability of target SINR achievability monitoring, active links can adaptively adjust their choice of a_i, b_i within the achievability bounds. If the estimate of λ_1 is observed to approach 1, links can relocate the transmission into another channel with a much lower value of λ_1. If a condition in which target SINRs cannot be achieved occurs, active links can either adaptively decrease their own target SINRs (e.g., to enable the medium access of the new link or reflect the changes in the topology) or migrate to another channel.

C. Optimum Medium Access

Medium access of inactive links or migrating active links is slightly more involved than monitoring of target SINR achievability. The reason for this is that the medium access mechanism must be founded in terms of *Theorem 1* on a reciprocal interaction throughout mutually caused interference by both the accessing links and the already active links.

We propose the implementation of medium access mechanisms using channel probing with variable transmit power. Before starting any data transmission, every accessing link i first transmits a predefined sequence of so-called channel probes [6] into the channel of interest. The only purpose of the probing signals is to create a well-defined additional interference in the tested channel. Different channels must be probed separately, but no additional overhead is required. Unlike in the probing schemes of [5], [6], [10], the TX power $\sigma_i(k)$ of each probe in time k is determined on the basis of the current level of the co-channel interference from the already active links (and other probing links) using the formula (1).

The parameters a_i and b_i of each probing link i are set to the value of the target SINR to be tested for achievability.

Similar to the already active links, the probe power is then used directly as the input for computing $\mathscr{W}_{\boldsymbol{n}}(k,t)$ and determining the achievability of the preferred target SINR. In case of collective computing $\mathscr{W}_{\boldsymbol{n}}(k,t)$, the extended input data can be gathered from both probing and already admitted links. Probing is preferably carried out on dedicated pilot frequencies, typically used in cellular systems.

V. Distributed Suboptimum Algorithm For Non-Iterative Channel Monitoring and Non-Invasive Medium Access

The method of *Theorem 1* enables the implementation of schemes for optimum management of transmit power and channel resources. Nevertheless, its decision making is not instantaneous as in the case of CSMA/CA because of the iterative computation of $\mathscr{W}_{\boldsymbol{n}}(k,t)$. Even though only a few iterations are typically sufficient for making a correct decision, such feature may be undesirable in applications with delay constraints on call search and selection (e.g., the 4G LTE system [1]). It is noteworthy that the *Assumption 1* never holds if only two links share one channel, i.e. $N = 2$ in *Theorem 1*. Furthermore, the proposed probing mechanism for implementing medium access causes additive interference, which may be undesirable for links on a tight energetic budget.

All these problems can be resolved by the following method which determines the SINR achievability from a single measurement of pilot signal strength. This low delay is realized at the cost of suboptimum performance of the scheme. If optimum performance is required, both the proposed methods must be combined.

The preliminary assumptions are largely relaxed:

Assumption 3 (On Existence of $\hat{\boldsymbol{\sigma}}$): It is assumed that $\boldsymbol{A}$ is invertible. □

Then the following theorem can be stated to generalize [9].

Theorem 2: Assume that a total of N active links transmit simultaneously in a shared channel while updating their TX powers based on (1). A unique equilibrium vector $\hat{\boldsymbol{\sigma}}$ from (2) exists and the power control algorithm (1) asymptotically converges to it, if the channel gains h_{ij} and parameters a_i are such that each link i fulfills

$$\max_{\forall i} (L_i) < 1, \tag{14}$$

where the set L comprises at least one of the following terms:

$$L_i^{(1)} = |a_i| \sum_{\forall j \neq i}^{N} \frac{h_{ij}}{h_{ii}}, \tag{15}$$

$$L_i^{(2)} = \sum_{\forall j \neq i}^{N} \left|a_j\right| \frac{h_{ji}}{h_{jj}} \tag{16}$$

□

Proof: Denote the i-th row of matrix $\boldsymbol{A}$ by $\boldsymbol{A}_{i*}$ and the j-th column of matrix $\boldsymbol{A}$ by $\boldsymbol{A}_{*j}$. Define also the p-norm of vector $\boldsymbol{z}$ as $\|\boldsymbol{z}\|_p = \left(\sum_{i=1}^{N} |z_i|^p\right)^{1/p}$ and $\|\boldsymbol{z}\|_\infty = \max_{\forall i} |z_i|$.

It is a well-known fact of matrix theory that $|\lambda_1| \leq \max_{\boldsymbol{z}\neq\boldsymbol{0}} \frac{\|\boldsymbol{A}\boldsymbol{z}\|_p}{\|\boldsymbol{z}\|_p} \forall p$. Furthermore, this inequality can be reduced to $|\lambda_1| \leq \max_{\forall j} \left\|\boldsymbol{A}_{*j}\right\|_1$ for $p = 1$ and to $|\lambda_1| \leq \max_{\forall i} \|\boldsymbol{A}_{i*}\|_1$ for $p = \infty$. Knowing that the element in the i-th row and j-th column of matrix $\boldsymbol{A}$ is given by $a_i h_{ij}/h_{ii}$ if $i \neq j$ and 0 if $i = j$, these inequalities reduce to the above ones (15) and (16). ■

We observe that the *Theorem 2* approximates the optimum criterion (3) by using an alternative criterion (14). This criterion is suboptimum since the approximation of λ_i by $L_i^{(1)}$ or $L_i^{(2)}$ may result in target theoretically feasible SINRs in terms of (3) being classified as infeasible. Nevertheless, the validity of (14) can be determined only on the basis of the knowledge of channel gains h_{ij} and parameters a_i.

Moreover, the evaluation of the term (15) can be carried out in a distributed manner. The parameter a_i is known to each link i; thus, links must only be able to determine $\sum_{\forall j\neq i}^{N} \frac{h_{ii}}{h_{ij}}$ to verify (15). This can be done by using a dedicated pilot signal, on which each active transmitter transmits with a constant predetermined power σ. To observe this, note that $\sum_{\forall j\neq i}^{N} \frac{h_{ii}}{h_{ij}} = \sum_{\forall j\neq i}^{N} \frac{h_{ii}\sigma}{h_{ij}\sigma} = \sum_{\forall j\neq i}^{N} \frac{h_{ii}\sigma}{I_i}$. In other words, the key term $\sum_{\forall j\neq i}^{N} \frac{h_{ii}}{h_{ij}}$ of link i equals to the ratio of (i) the pilot signal strength $h_{ii}\sigma$ from link i's own transmitter, and (ii) the value of interference I_i from other links $j \neq i$ in the pilot channel. Hence, the term $\sum_{\forall j\neq i}^{N} \frac{h_{ii}}{h_{ij}}$ corresponds to the actual SINR at which link i receives the pilot signal.

VI. Numerical Simulations

To illustrate the accuracy and convergence rate of the method presented in *Theorem 1*, Fig. 1 shows, as a function of λ_1, the average number of threshold iterations $\hat{k}$ required for medium access decisions made at *any* time $k \geq \hat{k}$ on the basis of the criterion $|\mathscr{W}_{\boldsymbol{n}}(k,0)| < 1$ to be identical to optimum decisions (3). The graphs represent both single link estimation of λ_1 ($\boldsymbol{n} = i$) and collective estimation by all active links ($\boldsymbol{n}$ comprises all i). The simulated network area is circular with a diameter of 1000 m, the user distribution is uniformly random, and the link length ranges from 100 to 150 m. The target SINR is set at 9.5 dB ($a_i = 8.9125$, $b_i = 0$), which is necessary to decode BPSK modulation with a bit error rate of 10^{-5} at a transmission range of 150 m. The channel gains exhibit an exponential path loss with an exponent 4. The noise level n_i is −108 dBm.

We observe that when $|\lambda_1|$ is relatively lower than the threshold value 1, both the individual and collective medium access can make correct medium access decisions anytime starting with already the very first sample $\mathscr{W}_{\boldsymbol{n}}(k,0)$), since $\hat{k} = 1$. The value of $\hat{k}$ increases when $|\lambda_1|$ is closer to 1. We can expect the average $\hat{k}$ to be up to ten under real-life networking scenarios as the operation of networks very close to saturation ($|\lambda_1| \sim 1$) is not desirable (small stability margin and the use of very high equilibrium TX powers $\hat{\boldsymbol{\sigma}}$).

Further, we compared the performance of both proposed methods from *Theorem 1* and *Theorem 2* (condition (15)) with standard random access, carrier sensing, and the optimum scheme (3). In particular, we examined the number of links in

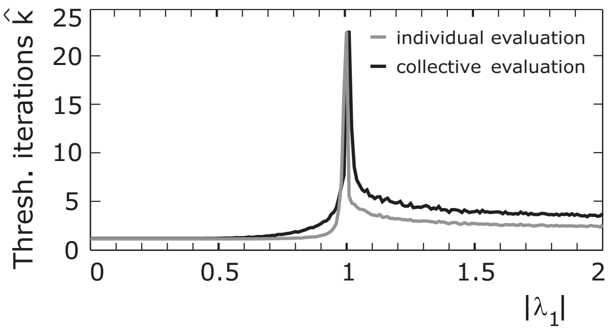

Fig. 1. The average number of threshold iterations $\hat{k}$ such that medium access decisions made at *any* time $k \geq \hat{k}$ on the basis of the criterion $|\mathscr{W}_{\boldsymbol{n}}(k,0)| < 1$ are identical to the optimum decision (3). The data are shown as a function of λ_1 and represent (i) single link estimation of λ_1 and (ii) collective estimation by all active links.

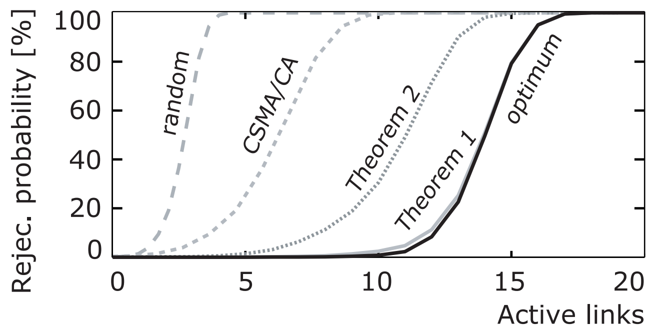

Fig. 2. Probability of network link rejection as a function of the number of simultaneously active links with SINR $\geq$ 9.5 dB. The proposed schemes from *Theorems 1* and *Theorem 2* (condition (15)) are compared with standard random access, carrier sensing, and the optimum scheme (3).

a random network that each of the above methods allows for simultaneous transmission in a single shared channel with a required SINR $\geq$ 9.5 dB.

Fig. 2 shows the probability of link rejection as a function of the number of simultaneously active links, assuming the above network setup and $\alpha = 3$. The graphs are plotted by integrating the frequency count histograms on the basis of 10^4 random topologies. The simulation of each topology was stopped after 10 consecutive links were rejected. Topologies with the lowest amount of collisions were considered to represent in case of random access and CSMA/CA algorithms.

We observe that the proposed scheme outperforms the comparison methods by admitting more links to transmit simultaneously in the shared channel, whereby its performance is basically identical to the genie-aided decision based on (3).

VII. Conclusion

We have proposed distributed adaptive algorithms for joint medium access and power control which allow multiple network links to simultaneously transmit over shared channels while maintaining a desired target SINR. Analogous to the standard carrier-sensing approach, the algorithms have low complexity and minimize communication overhead by using only simple interference measurements as their decision-making input. Only a few elementary arithmetic operations are required for internal data processing. More specifically, we have proposed a distributed iterative algorithm for optimum management of the energetic and channel resources of the network. Optimality is achieved in the sense that only links with achievable target SINR are admitted to communication and the equilibrium transmit powers are the lowest to satisfy the given target SINRs. The algorithm can be practically implemented by using channel probing. Furthermore, we have proposed a distributed suboptimum algorithm for instantaneous medium access in applications with delay constraints. This algorithm requires only to measurement of the strength of a pilot signal on a dedicated signaling channel. Advantageously, both algorithm types provide rejected links with a feedback on achievable target SINRs for readmission purposes.

References

[1] (2009) Technical specifications and technical reports for a UTRAN-based 3GPP system. [Online]. Available: http://www.3gpp.org/ftp/Specs/html-info/21101.htm

[2] A. Goldsmith, *Wireless Communications*. Cambridge University Press, 2005.

[3] H. Zhu, M. Li, I. Chlamtac, and B. Prabhakaran, "A survey of quality of service in IEEE 802.11 networks," *IEEE Trans. Wireless Commun.*, vol. 11, no. 4, pp. 6–14, Aug. 2004.

[4] G. Foschini and Z. Miljanic, "A simple distributed autonomous power control algorithm and its convergence," *IEEE Trans. Veh. Technol.*, vol. 42, no. 4, pp. 641–646, Nov. 1993.

[5] Z. Chenxi and M. S. Corson, "A distributed channel probing scheme for wireless networks," in *Proc. of IEEE INFOCOM 2001*, Anchorage, USA, Apr. 22–26 2001, pp. 403–411.

[6] N. Bambos, S. C. Chen, and D. Mitra, "Channel probing for distributed access control in wireless communication networks," in *Proc. of IEEE Globecom 1995*, Singapore, Nov. 13–17 1995, pp. 322–326.

[7] N. Bambos, S. C. Chen, and G. J. Pottie, "Channel access algorithms with active link protection for wireless communication networks with power control," *IEEE/ACM Trans. Networking*, vol. 8, no. 5, pp. 583–597, Oct. 2000.

[8] A. Halanay and V. Răsvan, *Stability and Stable Oscillations in Discrete Time Systems*. Amsterdam: Gordon and Breach Science Publishers, 2000.

[9] S. Kucera, S. Aïssa, and S. Yoshida, "Adaptive channel allocation for enabling target SINR achievability in power-controlled wireless networks," to appear in *IEEE Trans. Wireless Commun.*, 2010

[10] M. Xiao, N. B. Shroff, and E. K. P. Chong, "Distributed admission control for power-controlled cellular wireless systems," *IEEE/ACM Trans. Networking*, vol. 9, no. 6, pp. 790–800, Dec. 2001.

[11] S. Kucera and B. Zhang, "Delay analysis of a distributed scheme for optimized medium access control in power-controlled networks," in *Proc. of The Fifth IEEE Conference on Mobile Ad-hoc and Sensor Networks (MSN) 2009*, Wu Yi Mountain, China, Dec. 14–16 2009.