Optimum Allocation of Energy and Spectrum in Power-Controlled Wireless Networks with QoS Constraints

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***Abstract*—An important performance measure in wireless networks is the manner in which the network can distributively manage its limited energy and spectrum resources, while assuring certain quality of service for communicating users. The current practice is to develop schemes with low complexity that are based on workable, but theoretically suboptimal techniques such as random access or carrier sensing. To address the need for equally simple, but *optimally* performing resource management schemes, we propose a set of optimum distributed algorithms for adaptive admission control and power control, which jointly (i) maximize the number of transmitters that transmit simultaneously in shared channel(s) with a guaranteed target signal-to-interference and noise ratio (SINR) at the receiver; (ii) minimize the transmit powers required to satisfy the SINR targets; (iii) use interference measurements as the only decision-making input; and (iv) provide inadmissible links with feedback on feasible SINR for (re)admission purposes. Unlike previous studies in which SINR targets were assumed as constants, we defined these targets using arbitrary linear functions of interference. From numerical simulations, it is confirmed that the proposed scheme outperforms other schemes by achieving the theoretical performance bounds.**

***Index Terms*—resource management, optimum, distributed, adaptive, power, admission control, channel probing, eigenvalue**

I. Introduction

A conventional approach to the trade-off between efficient allocation of energy and spectrum and the need for certain guaranteed quality of service (QoS) in wireless networks is based on the separation of individual transmissions. This separation is achieved by using multiple access techniques that are based on the division of time, frequency, or code. However, the conventional approach technically requires network-wide synchronization and spreading code distribution; these cannot be easily realized in large mobile networks with distributed control and low-cost terminals.

In this study, we allow communication channels to freely shared by network links (transmitter-receiver pairs); this helps to avoid said technical issues, promotes spectral efficiency, and reduces communication delay. Our objective is then to analyze the challenging problem of realizing *optimum* and *low-complexity* management of the limited battery energy and spectral bandwidth, which guarantees a predefined level of QoS at all times under various/random operating conditions.

The constraints of optimality and low complexity have been assigned equal importance in our design. On the other hand, previous studies realize some of the design constraints at the cost of others Section II. The term "optimum" refers to a scheme that maximizes the number of transmitters that transmit simultaneously in a single channel (spectral optimality), and minimizes the transmit powers (energetic optimality) in order to maintain a predefined signal-to-interference and noise ratio (SINR) at each receiver (QoS optimality).

The term "complexity" primarily refers to the complexity of gathering input data for internal decision-making since any communication overhead directly affects the overall performance of wireless networks. "Low" complexity signifies that the input data can be obtained through simple distributed measurements analogically to the widely employed carrier-sensing techniques (e.g., CSMA/CA admission control mechanisms in the IEEE 802.11 standards [1] and standard power control loops with hard SINR constraints [2], [3]). The complexity in processing the input data is of secondary importance considering the low cost and high performance of modern microprocessors. In any case, the proposed solutions involve only a few arithmetical operations for data processing.

Our system model in Section III allows network links to express their QoS needs/preferences using the notion of an individual target SINR. Our motivation is the fact that the relative strength of the useful signal with respect to the interfering one(s) plays a key role in equalization, modulation, (de)coding, etc. [4]. More specifically, the target SINR is defined by an arbitrary linear (non-negative) function, which assigns transmit power values to the values of received co-channel interference. This feature generalizes the system models of related works. The target SINR must be achieved for each link in the shared channels at all times to reduce the probability of QoS outages under time-varying or random network conditions. We thus assume that the transmit power of each active transmitter is adapted and periodically adjusted in accordance with the local interference measurements of the corresponding receiver and the shared target SINR definition.

As follows from *Lemma 1* in Section IV-A, the transmit powers that the enable *simultaneous* satisfaction of a given set of SINR targets exist and can be allocated *if and only if* the absolute value of the dominant eigenvalue of the Foschini-Miljevic matrix $\boldsymbol{A}$ is smaller than one. The elements of $\boldsymbol{A}$ are defined on the basis of the SINR targets of all the network nodes and the actual channel gains between the nodes.

For this reason, we first propose an efficient distributed method for *exact* computation of the dominant eigenvalue of the Foschini-Miljevic matrix $\boldsymbol{A}$ under *varying* network conditions (Section IV). The method involves the use of simple

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interference measurements as the input and is applicable to both admission-seeking and already active links.

The real-time ability to distributively track down the variations of the dominant eigenvalue of $\boldsymbol{A}$ allows us subsequently to solve the problem of ensuring optimum joint energy and spectrum allocation in wireless networks with shared channels (Section V). The solution is expressed as low-complexity distributed algorithms; their iterative convergence is at an exponential speed, and they have theoretically optimum performance. Theoretically optimum performance implies that (i) *all* the links with unfeasible SINR requirements are rejected, and (ii) *all* the admissible links are accepted and allocated the *minimum* necessary transmit powers required to fulfill the SINR constraints. On a practical level, network links are required to measure local interference measurements and use it as a decision-making input in a manner similar to the simple CSMA/CA approach of the IEEE 802.11 DT function. Admission-seeking links also use a short sequence of channel probes with variable transmit power to accompany the initial interference measurements.

Numerical simulations in Section VI confirm that the proposed solution outperforms the prior art techniques such as CSMA/CA or [5]. The conclusions of our study are provided in Section VII.

II. Review of Previous Work

In the earlier studies, adaptive power control schemes such as [2], [6] were intuitively combined with inefficient and unscalable fixed channel assignment schemes. Other studies proposed the adoption of dynamic channel assignment based on random access (e.g. ALOHA) or carrier sensing and interference threshold evaluation (e.g, IEEE 802.11 DCF) to implement energy and spectrum allotment under QoS constraints. Such solutions are scalable and have low complexity, but suffer from a far-from-optimum performance in comparison to the mentioned optimum eigenvalue-based criterion.

Unfortunately, the computation of the network matrix eigenvalues is a difficult task. According to the Abel-Ruffini theorem, general closed-form formulae do not exist for networks with more than four links. Alternative numerical methods require unrealistic knowledge of the whole matrix $\boldsymbol{A}$ or its inverse. Therefore, recent studies have avoided eigenvalue computation by using the structures of some application-specific features to convert the above optimum eigenvalue-based criterion into a related and/or approximate criterion. In these studies, resource management schemes that involve rough approximations of the dominant eigenvalues of $\boldsymbol{A}$ were proposed. The schemes with the lowest input complexity use channel gain measurements [5] or local interference measurements [7], [8], which are obtained after transmission by one/multiple channel probe/s with constant power (proposed in [9]), as the input.

The performance of [5], [7], [9] is far from optimal since a large number of admissible passive links may be unnecessarily rejected. The scheme of [8] can in theory admit all admissible links, but practically suffers from protracted decision-making delay and is in a sense subject to the stop time problem [10]. Delay and energy dissipation are inherent in [11], which involves slow power-up and use of watchdog-type mechanisms for energy and spectrum allocation.

In the above mentioned studies, the links, once admitted, do not receive any subsequent feedback on power-control feasibility conditions and its variations under mobility or after new admission events in the given channel.

III. System Model and Its Properties

Consider with no loss of generality, a wireless network with a single frequency band for accommodating multiple transmissions from mobile users, who are arbitrarily distributed over the network area. Data from the upper layers of an active link are buffered and encapsulated into a continuous session at the MAC layer. The resulting series of data frames is transmitted periodically over a channel, which is represented by a combination of time and the available frequency spectra. We define h_{ij} to represent the channel gain between the transmitter of link i and the receiver of link j.

Due to channel sharing, every active receiver generally intercepts the desired signal from its corresponding transmitter and unwanted signals from at least one interfering transmitter. The level of co-channel interference $I_i = \sum_{\forall i \neq j} \sigma_j + n_i$ fluctuates over time due to network mobility, large-scale shadowing and/or admission events (n_i is the additive noise). Therefore, the transmitter of link i must periodically update its transmit power σ_i such that the actual signal-to-noise and interference ratio $\mathrm{SINR}_i = \frac{h_{ii}\sigma_{ii}}{\sum_{\forall i \neq j} \sigma_j + n_i}$ at the RX of link i is maintained above a predefined threshold level that satisfies the QoS of link i. Transmit data rate may be optionally adapted on the basis of the achieved SINR_i.

Formally, we assume that the transmit power σ_i is updated in time k as follows:

$$\sigma_i (k+1) = \frac{b_i + a_i I_i (k)}{h_{ii}} \qquad a_i, b_i \in \Re \tag{1}$$

This updating is based on the value of RX interference $I_i (k)$ in time k. In other words, the receiver SINR is maintained at $\mathrm{SINR}_i = a_i + b_i / I_i$.

Feedback of I_i from RX to TX is required to update the transmit power on the basis of Eq. (1). This feedback can be implemented by including interference measurement data into RX acknowledgments of TX data reception.

Adaptive variations of a_i and b_i help to ensure that the required QoS standards of link i are satisfied. Nevertheless, both the parameters should be varied with at speeds lower than the convergence rate of the processes, which are discussed below. In the case of QoS-oriented links that require constant SINR regardless of the interference, one can choose $b_i = 0$ and $a_i > 0$. Users who aim for optimum energy consumption can choose $b_i > 0$ and $a_i < 0$; this enables bursty transmissions with (i) high SINR and high data rate in the case of low co-channel interference, (ii) low power and low data rate in the case of high co-channel interference, and (iii) CSMA/CA-type interference threshold $I_i^{\max} = -b_i / a_i$ to avoid the wastage of energy under very high co-channel interference.

To understand the dynamics of the periodical adjustments of σ_i resulting from (1) by all the links i in a given channel, we define a vector $\boldsymbol{\sigma}$. The i-th component of this vector is equal to σ_i. The system attains the equilibrium $\hat{\boldsymbol{\sigma}}$ if

$$\hat{\sigma}_i(k) = \hat{\sigma}_i(k+1) = \frac{b_i + a_i\left(\sum_{\forall i \neq j} h_{ij}\hat{\sigma}_j(k) + n_i\right)}{h_{ii}} \quad \forall i. \quad (2)$$

Considering the notion of $\boldsymbol{\sigma}$, we observe that the above equality corresponds to the following:

$$\hat{\boldsymbol{\sigma}} = \boldsymbol{B} + \boldsymbol{A}\hat{\boldsymbol{\sigma}} \quad (3)$$

Here, the i-th component of vector $\boldsymbol{B}$ is $(b_i + a_i n_i)/h_{ii}$, and the element in the i-th row and j-th column of matrix $\boldsymbol{A}$ is given by $a_i h_{ij}/h_{ii}$ if $i \neq j$ and 0 if $i = j$. The matrix $\boldsymbol{A}$ will be henceforth called the Foschini-Miljevic matrix [2].

In game-theoretical terminology, the vector $\hat{\boldsymbol{\sigma}} = -(\boldsymbol{A} - \boldsymbol{E})^{-1}\boldsymbol{B}$ corresponds to a Nash equilibrium, where $\boldsymbol{E}$ is a unit matrix of the same size as $\boldsymbol{A}$. This equilibrium is Pareto optimal; in other words, there exists no other vector $\hat{\boldsymbol{\sigma}}$ with smaller components, which would satisfy all SINR_i for given a_i and a_i $\forall i$. The next *Lemma* states the necessary and sufficient conditions for the existence of $\hat{\boldsymbol{\sigma}}$, and its determination by power control algorithms (1).

Lemma 1 (Conditions for Power Control Feasibility): Optimum transmit powers $\hat{\boldsymbol{\sigma}}$ as defined in (3) exist and can be allocated by the power control algorithm (1) if and only if the channel is shared by concurrently active links, whose channel gains h_{ij} and parameters a_i are such that $\boldsymbol{A}$ has full rank, and

$$\max_{\forall i} |\lambda_i(\boldsymbol{A})| < 1, \quad (4)$$

where $\lambda_i(\boldsymbol{A})$ denotes the i-th eigenvalue of matrix $\boldsymbol{A}$. □
See e.g. [2] for proof.

IV. Distributed Computation of Dominant Eigenvalue of Foschini-Miljevic Matrix $\boldsymbol{A}$

A. Preliminary Observations and Prerequisites

According to *Lemma 1*, each channel in the network can be occupied by only those links whose SINR threshold throughout a_i validates Eq. (4). If this can be ensured by using the network admission control mechanisms, we can achieve optimum energy management. In other words, all the links are allocated minimum possible transmit powers at all times for the global satisfaction of their SINR requirements. This helps minimize the related energy expenditures under the given network conditions. Note that the state of power control feasibility is independent of b_i for any i.

Nevertheless, *exact* knowledge of the dominant eigenvalue $\max_{\forall i}|\lambda_i(\boldsymbol{A})|$ is necessary to implement optimum spectrum management, i.e., the optimum allocation of *both* spectrum and energy. Else, an incorrect admission control decision or setting of a_i may cause power control divergence and the related wastage of energy and QoS disruption.

Several observations on *Lemma 1* have provided sufficient basis to develop a technique to calculate the dominant eigenvalue by using network links with arbitrary precision (as discussed in the next subsection). First of all, the requirement of $\boldsymbol{A}$ having a full rank can be assumed to be practically valid, especially in a random (stochastic) environment [12], [13]. Second, numerical simulations of real-life networking topologies with more than two links revealed that the dominant eigenvalue of the corresponding matrices $\boldsymbol{A}$ are real numbers with algebraic multiplicity one.

The following *Assumption* summarizes these observations for formal purposes. The full rank requirement is replaced by the practically equivalent requirement that $\boldsymbol{A}$ be diagonizable.

Assumption 1 (On dominant eigenvalue of $\boldsymbol{A}$): Denote by λ_i the N eigenvalues of $\boldsymbol{A}$, indexed decreasingly by the order of magnitude. Then it is assumed that $\boldsymbol{A}$ is diagonizable, and λ_1, i.e., the dominant eigenvalue of $\boldsymbol{A}$, is such that $|\lambda_1| > |\lambda_{i\neq 1}|\, \forall i \neq 1$. □

B. Key Idea

At this stage, we demonstrate the key idea of our approach to solve for distributed computation of λ_1. For clarity, assume a simplified scenario: each link i targets a constant SINR_i independently of interference, and the additive noise n_i is neglectable compared to interference. In other words, $\boldsymbol{B} = \boldsymbol{0}$ (the next subsection addresses the general case of $\boldsymbol{B} \neq \boldsymbol{0}$).

Then the dynamics of transmit power allocation (1) is expressed as $\boldsymbol{\sigma}(k) = \boldsymbol{A}\boldsymbol{\sigma}(k-1) = \boldsymbol{A}^k\boldsymbol{\sigma}(0)$. The diagonizability of $\boldsymbol{A}$ from *Assumption 1* makes it possible to decompose the initial power vector $\boldsymbol{\sigma}(0)$ into $\boldsymbol{\sigma}(0) = \sum_{i=1}^{N} c_i\boldsymbol{x}_i$ for some constants $c_i \in \Re$. This is done by using the N eigenvectors $\boldsymbol{x}_i$, associated with each eigenvalue λ_i, as a basis. Since it holds that $\boldsymbol{A}\boldsymbol{x}_i = \lambda_i\boldsymbol{x}_i \forall i$, we obtain

$$\boldsymbol{\sigma}(k) = \boldsymbol{A}^k\boldsymbol{\sigma}(0) = \boldsymbol{A}^k\sum_{i=1}^{N} c_i\boldsymbol{x}_i = \sum_{i=1}^{N} c_i\lambda_i^k\boldsymbol{x}_i = \lambda_1^k\sum_{i=1}^{N} c_i\left(\frac{\lambda_i}{\lambda_1}\right)^k\boldsymbol{x}_i.$$

Since $|\lambda_1| > |\lambda_{i\neq 1}|$, it follows that $\lim_{k\to\infty}\left(\frac{\lambda_i}{\lambda_1}\right)^k = 1$ for any $i = 1$ and $\lim_{k\to\infty}\left(\frac{\lambda_i}{\lambda_1}\right)^k \neq 0$ for any $i = 1$. Consequently, if k tends to infinity, $\boldsymbol{\sigma}(k)$ tends to $c_1\lambda_1^k\boldsymbol{x}_1$ at an exponential rate. In other words, the vector $\boldsymbol{\sigma}$, composed of transmit powers of all network links, converges for $\boldsymbol{B} = \boldsymbol{0}$ to the $c_1\lambda_1^k$-multiple of the dominant eigenvector $\boldsymbol{x}_1$. This multiple is an eigenvector of $\boldsymbol{A}$ too, because $\boldsymbol{A}\left(c_1\lambda_1^k\boldsymbol{x}_1\right) = \lambda_1\left(c_1\lambda_1^k\boldsymbol{x}_1\right)$ as $\boldsymbol{A}\boldsymbol{x}_1 = \lambda_1\boldsymbol{x}_1$. However, if $c_1 \neq 0$, i.e., $\boldsymbol{x}_1^{\mathrm{T}}\boldsymbol{\sigma}(0) \neq 0$, it follows that

$$\lim_{k\to\infty}\frac{\boldsymbol{\sigma}(k+1)^{\mathrm{T}}\boldsymbol{\sigma}(k)}{\boldsymbol{\sigma}(k)^{\mathrm{T}}\boldsymbol{\sigma}(k)} = \lim_{k\to\infty}\frac{\boldsymbol{A}\boldsymbol{\sigma}(k)^{\mathrm{T}}\boldsymbol{\sigma}(k)}{\boldsymbol{\sigma}(k)^{\mathrm{T}}\boldsymbol{\sigma}(k)} = \lambda_1. \quad (5)$$

The last equality (5) shows that the term $\frac{\boldsymbol{\sigma}(k+1)^{\mathrm{T}}\boldsymbol{\sigma}(k)}{\boldsymbol{\sigma}(k)^{\mathrm{T}}\boldsymbol{\sigma}(k)}$ converges to λ_1. Recall from (1) that the input for calculating the values of $\sigma_i(k)\,\forall i$, i.e., term components, involves only the interference values $I_i(k)\,\forall i$. Thus, network links can optimally assess the feasibility of their SINR targets in the shared channel by simply iteratively evaluating the absolute value of the term $\frac{\boldsymbol{\sigma}(k+1)^{\mathrm{T}}\boldsymbol{\sigma}(k)}{\boldsymbol{\sigma}(k)^{\mathrm{T}}\boldsymbol{\sigma}(k)}$ based on their interference measurements. Owing to the exponential convergence rate, only few measurements (or iterations of (1)) are practically needed to obtain an estimate of λ_1 precise enough to make a correct decision on whether $\lambda_1 < 1$ or not in accordance with (4). Our simulations in [14] indicate that, depending on the network topology, typically up to ten iterations are sufficient.

C. Main Result

We have shown using (5) that *collective* measurements of interference in power-controlled networks provide sufficient information to compute the dominant eigenvalue λ_1 of the explicitly unknown matrix $\boldsymbol{A}$. However, before engaging in the practical design of optimum energy/spectrum management algorithms (next Section), several important technical issues have to be clarified:

- does a similar result hold in the *non-trivial* case $\boldsymbol{B} \neq \boldsymbol{0}$?
- would it then be possible for *individual* links i to compute λ_1 using only their own local interference measurements I_i, i.e., independently of other links $j \neq i$?
- can links use also interference samples, which were delivered with some non-zero and possibly varying but bounded *delay*, to obtain still accurate values of λ_1 ? Both in the case individual and collective computing cases ?
- will the exponential convergence speed and low input complexity remain *unchanged* in the above scenarios ?

The answer to all these questions is positive. The following theorem summarizes our results in more detail. Generally speaking, each single link i can, indeed, compute λ_1 by using interference measurements, which were collected from an arbitrary set n of links. The most practical choice is to use the measurements for individual links independently, in which case $n = \{i\}$. As apparent from the theorem proof, the convergence speed remains exponential. Moreover, if the input data $\boldsymbol{\sigma}_n(k)$ are delivered to a given link i with a delay of $t > 0$ update periods, the proposed computation method yields the value of λ_1^t instead of that of λ_1. This feature of our method is irrelevant, because $|\lambda_1| \gtrless 1$, when $\lambda_1^t \gtrless 1$.

Theorem 1: Consider a network based on the system model from Section III, in which N links i transmit in a shared channel, while updating their transmit powers based on the power control rule (1), and the *Assumption 1* is satisfied. Denote by $\boldsymbol{\sigma}_n(k)$ the vector of transmit powers of arbitrary n links in time k and define, for some bounded delays $t > 0$ and $x > 0$, the sequence

$$\mathscr{L}_n(k,t,x) = \frac{[\boldsymbol{\sigma}_n(k+t) - \boldsymbol{\sigma}_n(k+t-x)]^{\mathrm{T}} [\boldsymbol{\sigma}_n(k) - \boldsymbol{\sigma}_n(k-x)]}{[\boldsymbol{\sigma}_n(k) - \boldsymbol{\sigma}_n(k-x)]^{\mathrm{T}} [\boldsymbol{\sigma}_n(k) - \boldsymbol{\sigma}_n(k-x)]}. \tag{6}$$

Then, if

$$\boldsymbol{x}_1^{\mathrm{T}} \left(\boldsymbol{\sigma}(0) + \frac{\boldsymbol{B}}{\lambda_1 - 1} \right) \left(\lambda_1^x - 1 \right) \neq 0 \qquad \text{when } \lambda_1 \neq 1, \tag{7}$$

$$\boldsymbol{x}_1^{\mathrm{T}} \boldsymbol{B} \neq 0 \qquad \text{when } \lambda_1 = 1, \tag{8}$$

it holds that

$$\lim_{k \to \infty} \mathscr{L}_n(k,t,x) = \lambda_1^t. \tag{9}$$

Proof: The dynamics of the updating of the transmit power is based on (1) and can be expressed for $k \geq 0$ as follows:

$$\boldsymbol{\sigma}(k) = \boldsymbol{B} + \boldsymbol{A}\boldsymbol{\sigma}(k-1) = \boldsymbol{A}^k \boldsymbol{\sigma}(0) + \sum_{n=0}^{k-1} \boldsymbol{A}^n \boldsymbol{B}, \tag{10}$$

wherein $\boldsymbol{A}^0$ is a unit matrix. Thus,

$$\boldsymbol{\sigma}(k) - \boldsymbol{\sigma}(k-x) = \tag{11}$$

$$= \left(\boldsymbol{A}^k \boldsymbol{\sigma}(0) + \sum_{n=0}^{k-1} \boldsymbol{A}^n \boldsymbol{B} \right) - \left(\boldsymbol{A}^{k-x} \boldsymbol{\sigma}(0) + \sum_{n=0}^{k-1-x} \boldsymbol{A}^n \boldsymbol{B} \right) = \tag{12}$$

$$= \left(\boldsymbol{A}^k - \boldsymbol{A}^{k-x} \right) \boldsymbol{\sigma}(0) + \sum_{n=k-x}^{k-1} \boldsymbol{A}^n \boldsymbol{B}. \tag{13}$$

Assumption 1 assures thanks to its requirement of diagonizable $\boldsymbol{A}$ the existence of a linearly independent set of N eigenvectors x_i of $\boldsymbol{A}$ such that $\boldsymbol{A}\boldsymbol{x}_i = \lambda_i \boldsymbol{x}_i$ for any i. The N eigenvectors $\boldsymbol{x}_i$ can be therefore taken as a basis for expressing the initial power vector $\boldsymbol{\sigma}(0)$ as $\boldsymbol{\sigma}(0) = \sum_{i=1}^N c_i \boldsymbol{x}_i$ for some N constants c_i. Analogically, $\boldsymbol{B} = \sum_{i=1}^N d_i \boldsymbol{x}_i$ for some N constants d_i. Then the term $\boldsymbol{\sigma}(k) - \boldsymbol{\sigma}(k-x)$ can be rewritten as

$$\boldsymbol{\sigma}(k) - \boldsymbol{\sigma}(k-x) = \left(\boldsymbol{A}^k - \boldsymbol{A}^{k-x} \right) \boldsymbol{\sigma}(0) + \sum_{n=k-x}^{k-1} \boldsymbol{A}^n \boldsymbol{B} = \tag{14}$$

$$= \sum_{i=1}^N \left(c_i \lambda_i^k - c_i \lambda_i^{k-x} + \sum_{n=k-x}^{k-1} d_i \lambda_i^n \right) \boldsymbol{x}_i = \lambda_1^{k-x} \sum_{i=1}^N \gamma_i^x(k) \boldsymbol{x}_i, \tag{15}$$

where

$$\gamma_i^x(k) = \left(c_i \lambda_i^x - c_i + d_i \sum_{n=0}^{x-1} d_i \lambda_i^n \right) \left(\frac{\lambda_i}{\lambda_1} \right)^{k-x}. \tag{16}$$

Thus,

$$\mathscr{L}_n(k,t,x) = \tag{17}$$

$$= \frac{[\boldsymbol{\sigma}_n(k+t) - \boldsymbol{\sigma}_n(k+t-x)]^{\mathrm{T}} [\boldsymbol{\sigma}_n(k) - \boldsymbol{\sigma}_n(k-x)]}{[\boldsymbol{\sigma}_n(k) - \boldsymbol{\sigma}_n(k-x)]^{\mathrm{T}} [\boldsymbol{\sigma}_n(k) - \boldsymbol{\sigma}_n(k-x)]} = \tag{18}$$

$$= \frac{\lambda_1^{k+t-x} \left(\sum_{i=1}^N \gamma_i^x(k+t) \boldsymbol{x}_{i,n} \right)^{\mathrm{T}} \lambda_1^{k-x} \left(\sum_{i=1}^N \gamma_i^x(k) \boldsymbol{x}_{i,n} \right)}{\lambda_1^{k-x} \left(\sum_{i=1}^N \gamma_i^x(k) \boldsymbol{x}_{i,n} \right)^{\mathrm{T}} \lambda_1^{k-x} \left(\sum_{i=1}^N \gamma_i^x(k) \boldsymbol{x}_{i,n} \right)} = \tag{19}$$

$$= \lambda_1^t \frac{\sum_{i=1}^N \sum_{l=1}^N \gamma_i^x(k+t) \gamma_l^x(k) \boldsymbol{x}_{i,n}^{\mathrm{T}} \boldsymbol{x}_{l,n}}{\sum_{i=1}^N \sum_{l=1}^N \gamma_i^x(k) \gamma_l^x(k) \boldsymbol{x}_{i,n}^{\mathrm{T}} \boldsymbol{x}_{l,n}} \tag{20}$$

Observe that if $|\lambda_1| = \max_{\forall i} |\lambda_i| > |\lambda_j|$ for all $j \neq 1$ as required by *Assumption 1*, then $\frac{|\lambda_j|}{|\lambda_1|} = \left|\frac{\lambda_j}{\lambda_1}\right| < 1$. Consequently, $\lim_{k\to\infty} \left(\frac{\lambda_j}{\lambda_1}\right)^k = 0$ and $\lim_{k\to\infty} \gamma_j^x(k) = 0$ for any bounded x and $j \neq 1$. Hence, the limit (9) for $k \to \infty$ is equal to

$$\lim_{k\to\infty} \mathscr{L}_n(k,t,x) = \lambda_1^t \frac{\gamma_1^x(k+t) \gamma_1^x(k) \boldsymbol{x}_{1,n}^{\mathrm{T}} \boldsymbol{x}_{1,n}}{\gamma_1^x(k)^2 \boldsymbol{x}_{1,n}^{\mathrm{T}} \boldsymbol{x}_{1,n}} = \lambda_1^t, \tag{21}$$

because $\gamma_1^x(k+t) = \gamma_1^x(k) = c_1 \lambda_1^x - c_1 + d_1 \sum_{n=0}^{x-1} d_1 \lambda_1^n$.

The existence of (21) is guaranteed by the conditions (7) and (8). This is because their validity assures that $\gamma_1^x(k) \neq 0$, i.e., the denominator $\gamma_1^x(k)^2 \boldsymbol{x}_{1,n}^{\mathrm{T}} \boldsymbol{x}_{1,n}$ in (21) is not equal to zero. To prove this statement, we distinguish two cases for λ_1:

- In the first case $\lambda_1 \neq 1$, the term $\sum_{n=0}^{x-1} d_1 \lambda_1^n$ in $\gamma_1^x(k)$ can be expressed as the sum $d_1 \frac{\lambda_1^x - 1}{\lambda_1 - 1}$ of a geometric series with quotient λ_1, and we obtain

$$\gamma_1^x(k) = \left(\lambda_1^x - 1 \right) c_1 + d_1 \frac{\lambda_1^x - 1}{\lambda_1 - 1} = \left(\lambda_1^x - 1 \right) \left(c_1 + \frac{d_1}{\lambda_1 - 1} \right).$$

Therefore, the condition (7) corresponds to requiring $\gamma_1^x(k) \neq 0$.

- In the second case $\lambda_1 = 1$, it is true that $\lambda_1^x - 1 = 0$ and $\sum_{n=0}^{x-1} d_1 \lambda_1^n = d_1 x$, and we obtain $\gamma_1^x(k) = d_1 x$. Thus, $\gamma_1^x(k) \neq 0$ if $d_1 \neq 0$, i.e., if condition (8) holds.

■

V. Algorithms For Optimum Energy and Spectrum Management

The result of *Theorem 1* will be used in two ways: to provide a means to detect power control feasibility for the already active links, and to enable optimum admission control for entry-seeking (new-coming) links.

A. Active links

As seen from Eq. (9), every *active* link can easily track down the feasibility of its SINR target without any overhead. The optimum criterion (4) is implemented by a continuous evaluation of the sequence $\mathscr{L}_n(k,t,x)$ using the values of the periodically updated transmit power $\sigma_i(k)$. Clearly, $x+1$ data samples are needed before it is possible to compute the first element of this sequence. Numerical simulations confirmed that the orthogonality conditions (7) and (8) are practically satisfied as, roughly speaking, the vector $\boldsymbol{B}$ is never orthogonal to the dominant eigenvector $\boldsymbol{x}_1$. The time shifts $x > 0$ and $t > 0$ can be arbitrary. Only if λ_1 equals *exactly* to -1 for a long time, then the condition (7) implies that x must be an even number. However, this is unlikely in practice.

The ability to track down λ_1 in real time allows the active links to efficiently and safely adjust in a continuous fashion their QoS parameters a_i, b_i within the feasibility bounds. The exponential convergence speed of the algorithm guarantees a quick detection of any change of $\boldsymbol{A}$, i.e., λ_1, due to, for example, an admission event or variations of a_i, b_i by some link i. Detection of SINR-infeasibilities can result in a transmission of access-violation warning signals or justify the search for a communication channel with stable power control, if the infeasibility condition persist (the term passive links in the next subsection refers to such links as well).

In the simplest case of implementation of the proposed scheme, the input vector σ_n at each time k contains only the transmit power σ_i of link i ($n = \{i\}$). Preferably, however, the vector σ_n should contain not only σ_i, but also the powers $\sigma_{j\neq i}(k-x)$ of several other $n-1$ surrounding links $j \neq i$. That this measure increases numerical robustness of the algorithm is apparent from the proof of *Theorem 1* and is confirmed by numerical simulations in [14]. The additional data $\sigma_{j\neq i}$ can be gathered passively without any active data exchange and related overhead. The active links i would include information regarding their transmit power in their data/signalling packets. Other links j can then gather the necessary data σ_n by overhearing such packets.

B. Passive links

By definition, unlike active links, passive entry-seeking links are not engaged in any power control process. However, the feasibility of their SINR demands with respect to the chosen a_i and b_i has to be tested in given channel of interest before they engage in any full-fledged data transmission. This conflict can be resolved by allowing every admission-seeking link to transmit several channel probes with variable transmit power into the channel. These probes are basically testing signals [9], whose purpose is to cause some well-defined minor additional interference in the probed channel by imitating real data transmission (i.e., there is no need for actual data delivery).

The probe transmit power is calculated using consecutive local interference measurements and the formula (1). Then, the probing links can use directly the probe transmit powers to calculate the value of λ_1 from $\mathscr{L}_n(k,t,x)$ analogically to the already active links. Each channel of interest must be probed separately, yet no further overhead is necessary. If there are several candidate channels with $|\lambda_1| < 1$, it is preferable to choose the channel with the smallest $|\lambda_1|$ as the convergence of transmit power updates to the equilibrium is the fastest [15].

Owing to the use of probes, active links are able to track down the change in λ_1 due to the presence of new probing link(s). Moreover, the TX powers of the active links can be used to extend the input $\boldsymbol{\sigma}_n$ for calculating $\mathscr{L}_n(k,t,x)$ by the probing links and vice versa.

VI. Numerical simulations

We have compared the performance of the proposed algorithms for energy and spectrum allocation with two comparison schemes, namely, the standard CSMA/CA algorithm based on carrier sensing [1] and an algorithm from our previous study [5].

From a practical point of view, the input complexity of the both comparison schemes is equal to that of the proposed method. In our simulation, we have therefore focused on comparing the three schemes in terms of their performance. More specifically, we examined the number of links in a random network that each of the above methods allowed to transmit *simultaneously* in a single shared frequency band with some predefined SINR. Such a performance measure is directly related to the total capacity of the network, achieved by individual resource management methods.

For network simulation, we considered a circular network with 1 km radius and uniformly randomly distributed links. The link length is chosen randomly between 100 and 150 m. Channel gains h_{ij} are given by exponential path loss with exponent 5. Each link targets an SINR of 9.5 dB ($a_i = 8.9$), which is necessary to decode BPSK modulation with bit error rate of 10^{-5}. Noise power n_i is set to -108 dBm ($b_i = 1.585 \times 10^{-14}$). Several hundreds of different topologies were simulated, whereby the evaluation of each topology was terminated after 50 consecutive entry-seeking links were rejected. The sequence $\mathscr{L}_n(k,t,x)$ was evaluated for $n = \{i\}$ and $t = x = 1$.

Fig. 1 shows as a result a histogram of the number of links that were activated in the shared band with an SINR of at least 9.5 dB. We observe that the proposed scheme outperforms the other schemes by admitting more links. A numerical cross-check confirmed that the allocated equilibrium transmit powers

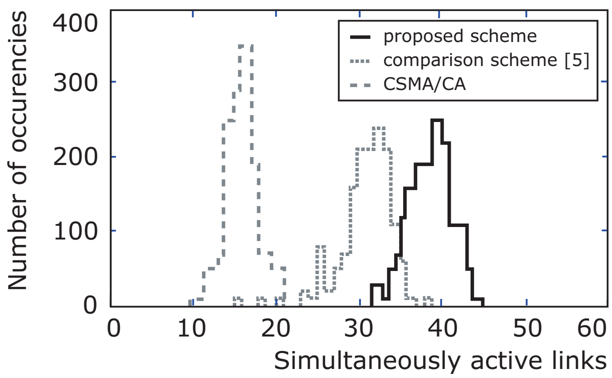

Fig. 1. Histogram of the total number of active links, sharing a single shared band with a guaranteed SINR of 9.5 dB. The proposed method (full line) is compared with a standard CSMA/CA algorithm [1] (dashed line) and a second comparison algorithm [5] (dotted line).

were the minimum necessary ones to satisfy the required SINR as they equaled to the theoretically anticipated values $\boldsymbol{\sigma} = -(\boldsymbol{A} - \boldsymbol{E})^{-1}\boldsymbol{B}$.

The large suboptimality of the CSMA/CA algorithm is due to SINR outages due to packet collisions. Links admitted by the algorithm [5] have a guarantee of achieving their SINR targets, but the algorithm is generally suboptimal. The theoretically optimum proposed scheme outperformed the comparison method [5] by approximately 30%. We have confirmed the anticipated optimality of our method by testing whether some of the rejected links could have been admitted by a genie-aided decision based on *Lemma 1*. All rejected links were inadmissible as expected. In other words, the proposed scheme admitted network links identically to the optimum decision-making by *Lemma 1*.

VII. Conclusion

We have proposed distributed adaptive algorithms for admission control and power control, which allow multiple network links to simultaneously transmit in a single channel while maintaining a predefined SINR. The algorithms manage the energetic and spectral resources of the network in an optimum way in the sense that the number of admitted links is the maximum possible one and the allocated transmit powers are the lowest necessary ones to satisfy given SINR targets. Advantageously, the algorithms are characterized by low input complexity thanks to the fact that they use simple interference measurements as their only decision-making input. Admission-seeking links accompany the initial interference measurements by short channel probing to determine the feasibility of their SINR requirement. Active links can track down the SINR feasibility conditions with no overhead at all.

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